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DECIMALS made Easie :

CONTAINING,

- I. An Explanation of Fractions in General, more especially Decimals, and how to Read or Write any Fraction whatsoever.
- II. Reduction, Addition, Subtraction, Multiplication, Division, and the Rule of Proportion in Decimals, with the Demonstration of each, and its Relation to the Rules for working Vulgar Fractions.
- III. The Reason why Decimals are wrought as whole Numbers.
- IV. The Excellency of Decimal Fractions above any other Fraction that can possibly be invented, proved by several Instances.
- V. The particular use thereof in Computing the Interest (Simple or Compound, or Discompt) of Money, and Rules for Purchasing or Selling an Estate, whether in Fee, or for Lives or Years, demonstrated in the Solution of the most useful Cases, with Proper Tables relating thereto.
- VI. The most easie Method of Extracting the Square and Cube-Roots of Numbers, whole or broken ; also the use thereof in solving several Geometrical Questions.

Wherein are many things never before made Publick.

Useful for Masters and Scholars ; also for Lawyers and Scriveners, or as a Foundation for any one that intends to acquire considerable knowledge in the Mathematicks by his own Industry.

By E. HATTON, Philomercat.

Nemo Arithmeticæ ignarus hic ingrediatur.

L O N D O N :

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Decimal Arithmetick.

CHAP. I.

An Explanation of Fractions in General, and how to Read or Write the same.

1. **A** Fraction is part of a Unit, and as no Number can possibly be so great, or contain so many Units but that it may be augmented or encreased by adding other Units thereto: so no Fraction can be so little, *i. e.* no Unit can be divided into so many parts but that those parts may still be subdivided or made less; so that the parts of a Unit are infinite as well as the number of Units are. *Vid. Le Grand's Philosoph. Part 2. Ch. 15.* *What parts a Unit may be divided into.*

2. All Fractions have 2 parts, *viz.* first the number of parts that the Unit is divided into, and secondly, That which shews how many of those parts a Fraction contains.

3. The number of parts that a Unit is divided into is called the Denominator, and is wrote under the Numerator: And the number of those parts contained in a Fraction is called the Numerator, and is wrote over the Denominator thus, in $\frac{3}{4}$; *What the Numerator and Denominator of a Fraction.*

$\frac{3}{4}$ = the Numerator.
4 = the Denominator.

So that this Fraction shews the Unit is divided into 4 equal parts, and that there are 3 of those parts in the Fraction; so is the Fraction to be read *three fourth parts*; and by the same Rule

$\frac{1}{2}$ of any thing is to be read One half.

$\frac{2}{3}$ is Two third parts.

$\frac{3}{4}$ is Three fourth parts.

$\frac{4}{5}$ is Four fifths.

$\frac{5}{6}$ is Five sixths.

$\frac{7}{8}$ is Seven eighths.

$\frac{9}{10}$ is Nine tenths of any thing.

And

An Explanation of Fractions in geneal.

And $\frac{3764}{10000}$ is 3 thousand seven hundred and sixty four ten thousandth parts; for in all Fractions you are first to read the Numerator and then the Denominator.

4. Of Fractions there are two distinct kinds, Vulgar and Decimal.

Of Vulgar Fractions there are 4 sorts, *i. e.*

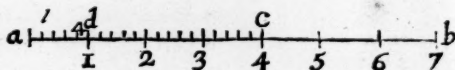
The several
kinds of
Fractions.

1. A Simple or single Fraction } Proper.
2. A Compound Fraction — } Proper.
3. A Simple Fraction } Improper.
4. A Compound Fraction } Improper.

5. A Simple Proper Fraction is any of those foregoing, which is immediately the Fraction of a Unit, as the Line *AB* being a Unit, or 1 divided into 7 e-

qual parts, the line

A. C. is 4 Sevenths, or $\frac{4}{7}$ of the said Line or Unit; and hence it



is plain, 1. That any number of Seventh Parts under 7 are Simple Fractions whose Denominators are 7. 2dly, That $\frac{7}{7}$, or where-ever the Numerator of a Fraction is equal to the Denominator, that Fraction is a Unit or One. But

A Com-
pound Fra-
tion, what.

6. A Compound Fraction is not immediately the the Fraction of a Unit as any of the 7th parts of the Unit or Line foregoing, but it is the Fraction of the Fraction (or part of another part) of a Unit, as the Line *a d* is $\frac{4}{7}$ of the Line *a b*, and the Line *a d* being $\frac{1}{7}$ of the Line *a b*, therefore the Line *a d* is $\frac{4}{7}$ of $\frac{1}{7}$ of *a b*, or a Unit, which $\frac{4}{7}$ of $\frac{1}{7}$ is a Compound Fraction.

Compound
may be re-
duced to
Simple Fra-
tions.

Hence it is evident, that a Compound Fraction may be reduced to a Simple, for if it were required to know what part of the Line *a b*, the line (*a d*) is; it must be considered that the line *a d* being divided into 5 parts, and that there is 7 times *a d* in the line *a b*, there is therefore 7 times 5 of the parts (*a d*) in the line *a b*, so that *a d* is $\frac{1}{35}$ part of the line (*a b*) and consequently the line *a d* is $\frac{4}{35}$ of the line *a b*, so that the Simple Fraction $\frac{4}{35}$ is equal to the Compound Fraction $\frac{4}{7}$ of $\frac{1}{7}$ of *a b*.

7. Having thus shewed you the Nature of a Simple and a Compound Fraction I come to distinguish between Proper and Improper Fractions.

An Explanation of Fractions in general. 3

A Proper Fraction is either a part or such a Number of parts of a Unit as is less than a Unit, so that the Numerator of this Fraction is always less than the Denominator, as $\frac{6}{7}$ or $\frac{34}{35}$, &c. of the line foregoing is a Proper Fraction thereof, because it is less than the whole line (*ab*). But

An Improper Fraction has the Numerator greater than the Denominator as $\frac{25}{7}$ of the line (*ab*) which is the whole line *ab* and the line (*ac*) added together, which being more than the line (*ab*) is improperly said to be a part thereof; so also $\frac{28}{7}$ of the line *ab* is an Improper Fraction containing just 4 times the line *ab*.

And hence it follows, that an Improper Fraction may be reduced to a whole Number, or to a whole Number and a Fraction, called a Mix'd Number: for in $\frac{28}{7}$ there is 4 Units, i. e. in 28 times (*ad*) there is 4 time (*ab*) which value of an Improper Fraction is always therefore found by dividing the Numerator by the Denominator.

Likewise from the Division of the line (*ab*) and what has been said, it is plain that one Fraction may be equal to another, and yet of different Numerators and Denominators, as the Fraction $\frac{20}{35}$ is equal to $\frac{4}{7}$, for the line *ab* being divided into 35 parts, (*ac*) is 20 of those parts, or if the line *ab* is divided into 7 parts, then the line (*ac*) is equal to 4 of those 7 parts; so that $\frac{4}{7}$ is the same value with $\frac{20}{35}$, because 20 contains 4 so often as 35 contains 7.

From whence it may be gathered, That of 2 Fractions, that is the greater value which has its Numerator nearest to the value of its Denominator, as I know that $\frac{16}{33}$ is of less value than $\frac{7}{11}$, for 7 contains 4 not twice, but 35 contains 15 above twice, so that $\frac{7}{11}$ or $\frac{35}{44}$ being a Unit (as was said in Paragraph the 5th foregoing) $\frac{16}{33}$ must be nearer a Unit, and consequently greater than $\frac{16}{33}$, for proof of which it is plain by the line (*ab*) above that the line *ac* being $\frac{4}{7}$, is more than the line (*a 3*) which is $\frac{16}{35}$, *Vide Demonstration of Division following.*

II. Having shewed how Fractions in general, whether Vulgar or Decimal may be read, and also so much of their Nature, as was necessary to the right understanding thereof; I shall next proceed to shew the property of *Decimal Fractions* more particularly.

A Decimal Fraction is any Fraction whose Denominator is 10 or 100, Definition or some Number that is a Unit, with Cyphers toward the Right hand: And they are called *Decimal Fractions*, because the Denominator is always either 10, or a Number produced by Multiplying 10 in it self continually, as 10, 100, 1000, 10000, 100000, &c.

B

2. By

2. By this definition, it is plain that the 2 last Fractions under Paragraph the 3d. *viz.* $\frac{2}{10}$ and $\frac{324}{1000}$, and such like are Decimal Fractions; so that though the nature of a Decimal Fraction with respect to its being a Fraction, be in effect the same with that of a Vulgar Fraction: Yet considering the nature of the Denominator, the working of Decimal Fractions is much different from that of Vulgar.

3. There is likewise a difference in the manner of writing Decimal Fractions from that of Vulgar, For though a Decimal Fraction be supposed to have a Denominator, yet that Denominator is seldom or never put down, For

Numeration of Decimals.

4. If it be considered (as was asserted in the first Paragraph of this general head) That the Denominator is always a Unit, with a Cypher or Cyphers toward the Right hand; it will be very easie (the variety being but little) to represent the Denominator of a Decimal Fraction by a point Comma, or other mark to be in the same place that the Unit of the Denominator would be, if the Fraction were wrote Vulgar Fraction-wise: Thus if $\frac{3}{10}$ or $\frac{46}{100}$ were to be expressed without their Denominators 10 and 100; it is easily done by prefixing a point to the Left hand, the 3 which is in the tens place, shews that the Denominator is 10, thus, .3; so likewise a point being in the third or hundreds place of 46, shews the 46 to be 46 hundred parts thus: .46; likewise $\frac{34}{1000}$ is wrote without the Denominator thus: .034, a Cypher being added to make the point possess the thousands place, because the Denominator is 1000; also $\frac{16}{10000}$ is wrote thus: .0016, two Cyphers being placed to the Left hand, that so the point may possess the tens of Thousands place, which shews that the Denominator is 10000.

How to read or write a Decimal.

5. Hence it is not at all difficult to read or write any Decimal without a Denominator by considering

1. That the Numerator is first to be read or written down as in whole Numbers, and

2. That the parts the Unit is divided into is always shewn by the place wherein the point stands, thus: .356 is three hundred fifty six thousand parts, *i. e.* The 356 being read, as in the Rules of Numeration of Whole Numbers, the Point standing in the thousands place shews the 356 to be 356 thousandth parts; and by the same Rule may any of the Decimals in the following Table be read.

C H A P. II.

Reduction of Decimals.

Why Reduction of Fractions is to be learnt first.

AS Vulgar Fractions must be first reduced to a common Denominator before they can be added, &c. so in Decimals Reduction is to be learnt before Addition, Subtraction, &c. because all Decimal Fractions of Money, Weight and Measures (Liquid and Dry) have their Original Derivation, and are produced with Vulgar Fractions; so that those Vulgar Fractions must first be reduced into Decimals before those Decimals can be added, Subtracted, &c.

One Vulgar Fraction reducible to another.

2. Vulgar and Decimal Fractions being of the same nature with respect to the Fraction compared to a Unit; it follows, that by the same Rule one Vulgar Fraction is Reducible to another, it is also Reducible to a Decimal.

3. Now one Vulgar Fraction may be reduced to another of the same value (though of different Numerators and Denominators) by this proportion.

As the Denominator of any given Fraction

Is to its Numerator;

So is any other Denominator whatsoever

To its Numerator;

which will make a Fraction equal to that given.

For instance, I would Reduce $\frac{3}{4}$ to a Fraction whose Denominator is 24; to find a Numerator to that Denominator, I say

Denom. Numer. Denom. Numer.

4. 3 :: 24. 18.

That is: As 4 (the Denominator given) is in proportion to 3, its Numerator: So is 24 the Denominator of the Fraction required, To 18 its Numerator, which $\frac{18}{24}$ is in value equal to $\frac{3}{4}$.

And by the same Rule, Vulgar Fractions, may be reduced to Decimals.

4. So that (as in the 2d. step above) if you would find the Numerators to Fractions whose Denominators are 10, 100, 1000, &c. which is all one with reducing a Vulgar to a Decimal Fraction, you may do it by the foregoing proportion.

For instance, To reduce $\frac{3}{4}$ to a Decimal Fraction whose Denominator is 100.

As

Reduction of Decimals.

7

As 4. 3 :: 100. .75

That is, As 4 is in proportion to 3 its Numerator ; so is 100 to 75 its Numerator, which is exprest without its Denominator 100, by placing a point in the hundreds place, as in the last Chapter is taught :

So is $\frac{3}{4}$ equal to the Decimal .75

And by the same Rule the Vulgar Fraction $\frac{1}{2}$ may be reduced to the Decimal $\frac{1}{10}$ or .5, or the Vulgar Fraction $\frac{1}{4}$ to the Decimal .25 for

First, As 2. 1 :: 10. .5

Secondly, As 4. 1 :: 100. .25, that is,

As 2 is to 1, so is 10 to .5, or as 4 is to 1, so is 100 to .25 ; so that

the Vulgar Fraction $\frac{1}{2}$, is equal to the Decimal .5 And

the Vulgar Fraction $\frac{1}{4}$, is equal to the Decimal .25, &c.

5. But it many times happens that the Numerator of the Fraction *What to do* given (when multiplied by 10, 100, 1000, &c.) cannot be divided *when re-* by the Denominator, without having a Remainder, in which case it *mainers* is the best way to Multiply the Numerator of the Fraction given by *happen in* 10000 or 100000, and divide that product by the Denominator *Dividing.* given, so shall the Decimal Fraction exhibited, be much nearer the truth, than if you had Multiplied the Numerator of the Vulgar Fraction given by 10, 100 or 1000, which Rule must always be observed, especially in producing Decimals, that are afterwards to be Multiplied by any whole Number, that consists of above 1 place.

1. For instance, If the Vulgar Fraction $\frac{3}{10}$ were to be reduced to a Decimal whose Denominator is 10:

The Decimal will be found $\frac{3}{10}$ or .4

2. If to a Decimal whose Denominator is 100.

The Decimal will be found .42

3. If to a Decimal whose Denominator is 1000.

The Decimal will be found .428

4. If to a Decimal whose Denominator is 10000.

The Decimal will be found .4285, &c

So that taking the $\frac{3}{10}$ for the Fraction of a Pound Sterling, if in reducing the same to a Decimal, you make the Denominator of that Decimal but 10, you omit or loose .0285 l. which is 6 $\frac{3}{4}$ d; if you make the Denominator of the Decimal but 100, you omit or loose .0085 which is 2d. and if you make the Denominator of the Decimal 1000 you omit, or loose .0005 which if to be multiplied but by 2, the loss by that omission will be near 1 Farthing.

6. Hence

What to do when nothing remains and all the dividend is not divided.

6. Hence it is evident that in reducing Vulgar Fractions to Decimals the safest way is to find a Numerator to a Denominator that is at least 10000; and if in dividing the product of the Numerator given Multiplied into 10000 by the Denominator given it happens that nothing remain before you have divided all the Figures in the Dividend, you need not divide any farther, but take the Quotient for the answer, and omit the Cyphers undivided in the Dividend; for instance, If you would reduce $\frac{1}{8}$ to a Decimal Fraction whose Denominator is 10000.

As 8. $1 :: 10000$

$$\begin{array}{r}
 \text{I} \\
 \hline
 8 \overline{) 10000} \quad (.125 \\
 \underline{0} \\
 20 \\
 \underline{0} \\
 40 \\
 \underline{0} \\
 0
 \end{array}$$

Here you see, That I make no use of the Cypher next the Right hand in the Dividend, because there is nothing remains when I have divided 1000; from whence I gather that in order to the true valuing the Figures in the Quotient, to know when you may, and when you may not put Cyphers towards the Left hand, the Figures therein observe this

General Rule.

Why Cyphers to the Right hand in a Decimal signifies nothing, which is a Rule for reducing a Decimal to its lowest Terms.

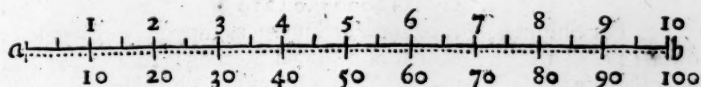
That as many Cyphers as are divided of those you assign for the Denominator of your Decimal Fraction, so many Decimal places must you have in the Quotient, and if there is not so many places in Figures, you must make up the number by prefixing Cyphers towards the Left hand: This Rule you will find useful in the Reduction of Coin, Weight and Measure to Decimals, as will appear by and by.

7. From the sixth or last step it is plain, That a Cypher or Cyphers next the Right hand in the Numerator of a Decimal does neither augment nor diminish the value of that Decimal, for otherwise the omitting

omitting the Cypher in the Units place of the dividend above would make the Quotient (125) too little: But since the places next the Right hand in the Denominator of a Decimal are always Cyphers, therefore Cyphers in the Numerator must be of no value, for $\frac{1000}{10000}$ is of no greater value than $\frac{100}{1000}$ or $\frac{10}{100}$, or $\frac{1}{10}$, and consequently the .125 above is of the same value with .1250.

To Demonstrate this yet more plain.

Suppose the line following be a Unit divided into 10 parts, and each of those into 10 makes 100 parts:



It is undeniably true, that the line (ab) being divided into 100 equal parts, the line (a20) which is 20 hundred parts of the line (ab) is equal to (a2) which is 2 Tenth parts of that line (ab) so 30 hundreds is no more then 3 Tenths, &c.

8. And as the same Rule serves both for reducing one Vulgar Fraction to another, and to a Decimal: so likewise may you reduce a Decimal to a Vulgar Fraction.

To reduce a Decimal to a Vulgar Fraction.

For Instance, If it were required to reduce .125 to a Vulgar Fraction whose Denominator is 16, what must be the Numerator? by the third step of this Chapter.

As 1000. 125 :: 16. 2.

So that $\frac{2}{16}$ is a Vulgar Fraction equal to the Decimal .125, and so much for Reduction in General, I proceed to shew next

I. How to reduce the parts of Coin, Weight and Measure to Decimals, and

II. To find the Value of any Decimal, whether the same be of Coin, Weight, &c.

9. First for reducing Farthings, Pence, or Shillings, or Shillings and Pence, Pence and Farthings, or Shillings Pence and Farthings into the Decimals of a Pound; you must first reduce the same to the Vulgar Fraction of a Pound, and then that Vulgar to a Decimal, by the third step of this Chapter.

A Rule to reduce Shil. Pence, &c. to the Decimal of a Pound.

For

For Instance, If you would reduce 9 d. into the Decimal of a Pound, consider that 9 d. is $\frac{9}{240}$ of a Pound for 1 Penny is $\frac{1}{240}$ of a Pound: then by the said third step, observing also the fifth and sixth,

As 240. 9 :: 10000. .0375

So that the 9 Pence being multiplied by 10000, and that product divided by 240, the Quotient is 375, and by the General Rule under the sixth step, is to have a Cypher before it, which is .0375, the true Decimal of 9 Pence.

10. For Pence and Farthings, as $10\frac{1}{4}$ Pence &c. reduce the Pence and Farthings into Farthings, as $10\frac{1}{4}$ d. is 41 Farthings; now 1 Farthing being $\frac{1}{960}$ of a Pound, therefore 41 Farthings is $\frac{41}{960}$, and consequently by the third step of this Chapter.

As 960. 41 :: 10000. .04270

11. For Shillings, Pence and Farthings; as suppose you would reduce 13 s. $6\frac{1}{4}$ to the Decimal of a Pound Sterling: In 13 s. $6\frac{1}{4}$ d. are 649 Farthings, so is 13 s. $6\frac{1}{4}$ $\frac{649}{960}$ of a Pound. Therefore

As 960. 649 :: 1000. .676

12. As for reducing any number of Shillings it may be done by the same Rules as in the 9th step, &c. first reducing the Shillings into the Vulgar Fraction of a Pound, and then working by the Rule of Proportion, as 6 s. is $\frac{6}{20}$ of a Pound, and as 20 is to 6, so is 100 to .30 the Decimal of 6 s. But the most concise way of Reducing any Number of Shillings to the Decimal of a Pound, is by this

Rule.

*A short way
for redu-
cing Skill.
to the De-
cimals of a
Pound.*

If the Shillings are an even Number, half thereof is the Decimal: or if they are an odd Number, put a Cypher toward the Right hand thereof; and then half that Number is the Decimal sought; thus:

The Decimal of $\begin{array}{l} s. \\ 2 \text{ — is —} .1 \\ 4 \text{ — — —} .2 \\ 6 \text{ — — —} .3 \\ 8 \text{ — — —} .4 \end{array}$ &c.

Also of — 3 or 30 is = .15 of a Pound Sterling.

$\begin{array}{l} 5 \text{ or } 50 = .25 \\ 7 \text{ or } 70 = .35 \\ 9 \text{ or } 90 = .45 \end{array}$ &c.

13. By

13. By the Rules above, any Number of Pence or Farthings under 1 s. are reducible to the Decimal of a Shilling, &c. as 3 d. being $\frac{3}{12}$ of a Shilling, the Decimal is .25 of a Shilling, so likewise 6 pence or $\frac{6}{12}$ is .5 of a Shilling.

Also 1 Farthing is $\frac{1}{4}$ of a Penny or .25 of a Penny in a Decimal; and $\frac{1}{2}$ Penny is .5 thereof in a Decimal, and so of the rest.

14. Having shewed how to reduce any Denomination of Money to *Ounces to the Decimal of a higher Denomination*, I shall next shew how to reduce the parts of weight to Decimals; and first of *Avoirdupoise of a Pound*. weight, for reducing any number of Ounces into the Decimal of a pound, as $7\frac{1}{3}$ which is $\frac{7}{16}$ of a pound, and therefore in a Decimal is .4375 for

As 16. 7 :: 10000. .4375 of a lb.

Also by the same Rule 3 Ounces, 8 Drams, or 56 Drams is $\frac{56}{256}$ lb, (256 Drams being a pound) which Vulgar Fraction in a Decimal is .21875 of a pound, for

As 256. 56 :: 100000. .21875

Also for reducing quarters of hundreds and pounds into the Decimals of a hundred weight of 112 lb, as 14 pound is $\frac{14}{112}$ of a hundred, and that in a Decimal is .125 C. for

As 112. 14 :: 1000. .125

Likewise 1 quarter 21 pound is by the same Rule .4375 C. for

1 q. 21 lb is 49 lb or $\frac{49}{112}$ of a C. And

As 112. 49 :: 10000. .4375 of a hundred.

15. For reducing *Troy Weight as Penny-weights, Grains, &c. to Troy* the Decimal of an Ounce, or a Pound, observe the said Rule for bringing first the same into Vulgar Fractions, and then those Fractions into Decimals. Thus 15 penny weight is $\frac{15}{16}$ of an Ounce, or .75 of an $\frac{3}{4}$, for

As 20. 15 :: 100. .75

Also by the same Rule 7 penny weight 12 grains, or 180 grains is $\frac{180}{480}$ of an Ounce, or in a Decimal, .375 $\frac{3}{8}$, for

As 480. 180 :: 1000. .375

And by the same Rules it is very easie for a mean Capacity to understand the reducing the parts of Liquid, or Long Measure into the Decimals of superior Denominations.

II. Thus having shewed how to reduce the parts of Coin, Weights, &c. to Decimals, I come now to shew the Method of finding the value of any Decimal, for which take this

General Rule.

*A Rule to
find the
value of
any Deci-
mal.*

Multiply the Decimal given by such a Number of Units of the next inferior Denomination, as is equal to a Unit of the Denomination your Fraction is of, and from the Right hand the product cut so many places as are contained in the Decimal given; so those Figures to the Left hand, such point or dash of Separation, are Units of the said next lower Denomination, and those to the Right hand are parts of one of those Units; Examples will Demonstrate this:

Example, 1. of Coin.

What is the value of .775 of a Pound Sterling?
See the Work according to the Rule.

Shillings in a Pound $\overset{.775}{= 20}$ } Multiply

Product 15. 500 s. }
Pence in a Shilling $\text{---} 12$ } Multiply

6.000 Product, Answer l. 00: 15: 06:

The Relation this Rule has to that for finding the value of a Vulgar Fraction.

2. The Rule to find the value of a Vulgar Fraction is, 1st. To Multiply the Numerator of the Fraction, by such a Number of Units of the next inferior Denomination, as is equal to a Unit of that Denomination your Fraction is of, which is the very same with the Rule above, the Decimal given being the Numerator of the Fraction given. 2dly, in Vulgar Fractions, the last product is to be divided by the Denominator of the Fraction, which is the same with cutting off Figures from the product, as in the Rule above; the Denominator of a Decimal being only 1 with Cyphers, as 10, 100, 1000, &c. by any of which Numbers another may be divided, by cutting as many figures from the Dividend, as there are Cyphers in the Divisor, so those figures toward the Left hand are the Quotient, and those to the Right hand the dash are the remainder.

3. But

3. But to make this more apparent, the Decimal Fraction above, viz. .775 in a Vulgar Fraction is $\frac{186}{240}$, by the 8th step of this Chapter, now 186 multiplied by 20, and divided by the Denominator 240, the Quotient is 15 s. as above, and the remainder 120 s. reduced into Pence by multiplying by 12, is 1440 Pence, which divided by 240, the Quotient is 6d. and nothing remains.

$$\begin{array}{r}
 \frac{186}{240} \text{ l.} \qquad \frac{186}{20} \left. \vphantom{\frac{186}{20}} \right\} \text{ Multiply} \\
 \hline
 240 \overline{) 3720} (15 \text{ Shilling} \\
 \underline{240} \\
 1320 \\
 \underline{1200} \\
 120 \left. \vphantom{120} \right\} \text{ Multiply} \\
 \hline
 240 \overline{) 1440} (6 \text{ Pence} \\
 \underline{240} \\
 1200 \\
 \underline{1200} \\
 0 \text{ remains.}
 \end{array}$$

Whereby it is evident, That whoever knows how to find the value of a Vulgar Fraction, may likewise by the same Rule find the value of a Decimal.

The Demonstration of the last General Rule for finding the value of a Fraction.

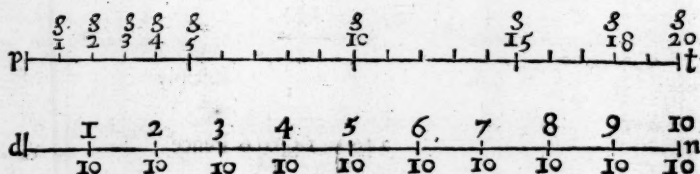
4. Admit the lines (*p t*) represent 1 Pound Sterling, divided into 20 equal parts, which parts will each represent 1 Shilling. Now suppose there is a given Fraction, whose value is required as .9 of a Pound: Draw another line of the same length with the former, and parallel thereto as the line (*D N*), which divide according to the nature of the Fraction, whose value you seek; as here into 10 equal parts according to your Denominator, and right against .9 of those parts, in this last line stands 18 Shillings in the line of Shillings, which shews that .9 of a Pound Sterling is in value 18 Shilling. And the like of any other Fraction.

Reduction of Decimals.

For, As 10 the whole line (*dn*) or Denominator of the Fraction given, is in proportion to 20 Shillings the whole line (*pr*): So is .9 the Numerator of the Fraction given (as in the lower line (*d* 9) to 18 Shillings, or the line (*p* 15) in the upper line, the value required; and therefore do you Multiply and Divide as in the Rules directed.

$$\begin{array}{r} 10. 20 :: 9 \\ 20 \end{array}$$

10) 18.0 Shillings the Answer.



And this Reason serves not only for Money, but also Weight, Measure, &c. being general as the Rule is.

A Rule to find the value of the Decimal of a Pound Sterling by inspection.

5. Double the Digit standing next the point in the Decimal given, and (if the next Figure toward the Right hand that be 5 or more) add 1 to the product; then for the figure standing in the said second place, reckon so many tens of Farthings, as the same is more than 5; or according to the figure if less than 5, and reckon the figure in the third place so many Farthings, which (as often as they are above 25) make less by 1.

The Reason of this Rule Demonstrated with an Example.

1. That place next the point in a Decimal Fraction, is called the *Prime's* place, because it is the first Quotient of the Unit divided by 10; so that any figure there, is so many tenth parts of a Unit; for
instance

instance in the Fraction .775 foregoing the 7, next the point is 7 tenths of a pound or 14 Shilling, and therefore does the Rule direct to double the figure in that place.

2. The Reason why the Rule says you are to add 1 to the last product when the figure in the second place from the point is 5, or more, is because 5 in that place, i. e. 05 is always 1 Shilling, because it is $\frac{5}{10}$ of $\frac{1}{10}$, or $\frac{1}{2}$ of $\frac{1}{20}$ of a pound, now $\frac{1}{20}$ of a pound being 2 s. $\frac{1}{2}$ of that must needs be 1 s. which 1 in the Example makes the former product 15 s.

3. As the place next the point is so many tenth parts, and the second from the point so many hundred parts of a Unit, or Tenth parts of $\frac{1}{10}$: So that figure in the third place from the point is so many tenth parts of $\frac{1}{100}$, or a thousand parts of a Unit, as 5 in the Example is 5 tenths of 1 tenth of 1 tenth of a Unit, which is 5 thousandth part of a Pound.

4. So that what is said in the Rule of calling the figure in the second place above, or under five, so many tens, and that in the third place from the point so many Farthings (as in the Example 25 Farthings), supposes 1000 Farthings in a Pound Sterling, and there being but 960 in a Pound, therefore the Rule must be something Erroneous. But that is rectified well enough, by deducting (for the 40 Farthings that 1000 exceeds 960), 1 at every 25; for if 1000 Farthings is 40 above 1 Pound, then 500 is 20 above, and 250, 10 too much, or 25 Farthings 1 too much.

5. So that in the Example of .775 l. the .7 is 14 s. and the .07 being above 5, the 14 s. must be made 15 s.

Then the 2 above 5 in the second seven from the point, being so many 10 Farthings is 20, and the 5 in the third place is 5 Farthings, which together is 25 Farthings, which by the 4th. step of this Demonstration being 1 too much, must be made 24 Farthings, or 6 d. so is the value (by inspection) of .775 l. = 15 s. 6 d. which is evident by the Example next following the last General Rule.

6. The same General Rule above serves also for reducing or finding the value of a Decimal, of *Avoirdupois* Weight, or *Troy* Weight, Measure, &c. as will appear in the Examples following.

Example 2.

What is the value of .3775 of a Hundred Weight, or 112 Pounds?

See

Reduction of Decimals.

See the Operation.

Multiply $\left\{ \begin{array}{l} .3775 \text{ of a Hundred} \\ 4 \text{ Quart. in a Hundred.} \end{array} \right.$

Multiply $\left\{ \begin{array}{l} \text{Quar.} = 1.5100 \text{ parts of a Quarter.} \\ 28 \text{ Pounds in 1 Quarter.} \end{array} \right.$

408
102

Multiply $\left\{ \begin{array}{l} \text{Pounds} = 14.2800 \text{ parts of 1 Pounds} \\ 16 \frac{2}{3} \text{ in 1 Pound} \end{array} \right.$

168
28

Ounces = 4.4800 parts of an Ounce
qu. lb $\frac{2}{3}$

So the Answer is 1: 14: 4 $\frac{48}{100}$ or $\frac{12}{25}$.

Example 3.

What is the value of .3125 of a Pound Troy? See the Work.

Answer 3: 15. $\frac{2}{3}$ dw. .3125 of a Pound
12 Ounces in 1 pound } Multiply

3.7500 product = 3 $\frac{2}{3}$.75 } Multiply
20 Penny weight in an $\frac{2}{3}$

15.0000 Penny weights.

Example 3.

Example 4.

What is the value of .5125 of a Barrel of Ale?

.5125 of a Barrel
4 Firkins in a Barrel } Multiply

Firkins = 2.0500 parts of a Firkin
8 Gallons in a Firkin } Multiply

.4000 parts of a Gallon, or $\frac{4}{10}$

Firkin	Gallon.
Answer 2:	00.4

C H A P. III.

Addition of Decimals.

IN Addition of Decimals there is no difficulty, observing, to place *Ruls.* the points, and places there from, one under another, and to cut from the Right hand the Summ as many places as are in that Decimal that has most places in those given. As in the several Examples following.

Example I.

To .4765 Add .073 and .1172
By the same Rule that Numbers of one Denomination are added the Summ of the 3 Decimals is, .6667 l. or 13 s. and 4 d. which is thus proved.

.4765	}	Add
.073		
.1172		
.6667	Summ Total.	

Parts:

		s.	d.
Parts of	{ .4765	is =	09 : 06 $\frac{1}{2}$
1 Pound	{ .073	is =	01 : 05 $\frac{1}{2}$
	{ .1172	is =	02 : 04 $\frac{1}{2}$
<hr/>		<hr/>	
Total = .6667		= 13 : 04.	

which 13 s. 4 d. is both the true Summ of the Shillings and Pence given, and also the value of the Decimal .6667.

Example 2.

To 376.04 Add .57983 and .051

$$\begin{array}{r} \text{Add } \left\{ \begin{array}{l} 376.04 \\ 0.57983 \\ 0.051 \end{array} \right. \\ \hline 376.67083 \text{ Total.} \end{array}$$

Example 3.

To 476 Add 32.9, and .87 and .775

$$\begin{array}{r} 476.0 \\ 32.9 \\ 0.87 \\ 0.775 \\ \hline 510.545 \text{ Summ} \end{array} \quad \left. \vphantom{\begin{array}{r} 476.0 \\ 32.9 \\ 0.87 \\ 0.775 \end{array}} \right\} \text{Add}$$

By observing the manner of placing and adding the Numbers in the 3 Examples above, you may easily add any Decimal Fractions whatsoever; so that I shall proceed to shew

The

The Relation the Rule for Adding Decimals has to that for Vulgar Fractions.

To add Vulgar Fractions or mixt Numbers together. The Rule is, To reduce the Fractions to a common Denominator, and then the Summ of the Numerators divided by the common Denominator, quotes the Summ or Answer.

So it is in Decimals, which have always a common Denominator; for as much as Cyphers to the Right hand of the Numerator of a Decimal does not alter its value by the 7th step of Chap. 2. So that the Decimal Fractions in the last Example in a Common Denominator stand thus: which are the

same with those above, the Denominator to that Fraction that has most places, being always the common Denominator, as here 1000, by which 2545 (the Summ of the Numerators) being divided the

.900

.870

.775

2.545 the Summ (as before) of the Fractions.

Quotient is 2.545 the true Summ sought, so that the Rule for adding a Decimal is the very same with that for adding a vulgar Fraction, but the operation for Decimals is much easier for 2 Reasons. 1. Because they need no reducing to common Denominators; and 2. Because the Denominator or Divisor whereby the Summ of the Numerators is divided, being 10, 100, or 1000, &c. divides any other Summ without Work, only by cutting off as many places toward the Right hand the Dividend, as there are Cyphers in the Divisor, as in the Example 2545 is divided by 1000, by only cutting off 545, so is the Quotient 2.545, or $2\frac{545}{1000}$.

The Demonstration of Addition of Fractions.

If the Summ of the Numerators and Denominators of Fractions would give the true result or aggregate, as if $\frac{2}{3}$ and $\frac{1}{4}$ were $\frac{5}{7}$, then this Rule would not seem so difficult as many fancy it: But they think it strange that the Summ of 2 or 3 Fractions cannot be known, without the trouble of first reducing the Fractions to one Denominator, and afterward dividing the Summ of the Numerators thereby: I shall therefore shew first why Fractions are first to be reduced to a common

D

Denominator;

Denominator ; and secondly why the Summ of the Numerators is to be divided thereby.

The necessity of reducing Fractions to a Common Denominator, before Addition.

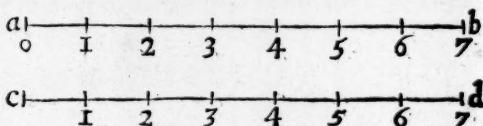
1. It is impossible to add Fractions that have different Denominators together, so as to give the true Total ; because the Unit in each Fraction being divided into a different number of parts, it cannot be expressed without reducing those different parts into one and the same, how the Summ of the Numerators bears proportion to the Denominators : But if the Denominators are alike, then the Summ of the Numerators is the Summ of the Fractions, if divided by one of the Denominators thus : $\frac{3}{7}$ and $\frac{5}{7}$ is $\frac{8}{7}$, because in both Fractions the Unit is divided into 7 parts, and the Summ of 3 and 5 parts in either

Line is 8 parts, which is one whole

Line and $\frac{1}{7}$ part more, and therefore is the Summ of the

Numerators always

to be divided by the Denominators, that so it may be known how many Units the Total of the parts contains : As in the Example above, likewise $\frac{4}{7}$ and $\frac{6}{7}$ is $\frac{10}{7}$, now to know how many times $\frac{1}{7}$ or 1, equal to the Line ab , is in $\frac{10}{7}$ the way must be to divide the Number of parts in the Summ of the Numerators (as here 10) by the number of parts that the Unit is divided into (which is here 7) and the Quotient shews the Units, and the remainder the parts of a Unit, as $\frac{3}{7}$ of the Line ab or cd is once the whole Line (or $\frac{7}{7}$) and 4 parts, or the distance from a to 4, or c to 4.



Why the Summ of the Numerators are to be divided by the Denominator.

I shall shew next before the use of Decimals, why Decimals are added as whole Numbers.

CHAP.

C H A P. IV.

Subtraction of Decimals.

Subtraction of Decimals is performed in every respect as Sub-*Rule.*
 straction of whole Numbers of one Denomination, observing the
 Rule given in Addition for the placing of the Decimals, and point-
 ing out the Decimal places of the remainder, which must be always
 the same with that of the most places of the Decimals given.

Example 1.

From .976 take .037647?

$$\begin{array}{r}
 \text{From} = .976000 \\
 \text{Take} = .037647 \\
 \hline
 \text{Rems} = .938353 \\
 \hline
 \text{Proof} = .976000 \text{ Summ.}
 \end{array}
 \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \text{Add}$$

Example 2.

From 376.4 take .0075?

$$\begin{array}{r}
 \text{From} \quad 376.4 \\
 \text{Take} = \quad .0075 \\
 \hline
 \text{Rems} \quad 376.3925 \\
 \hline
 \text{Proof} = 376.4 \quad \text{Summ}
 \end{array}
 \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \text{Add}$$

Subtraction of Decimals.

Example 3.

From 376 take .376

$$\begin{array}{r}
 \text{From} \quad 376.0 \\
 \text{Take} \quad \underline{.376} \\
 \text{Refts} = 375.624 \\
 \text{Proof} \quad 376 = \text{Summ}
 \end{array}
 \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Add}$$

Example 4.

From 376 take 1.47689

$$\begin{array}{r}
 \text{From} \quad 376 \\
 \text{Take} \quad \underline{1.47689} \\
 \text{Refts} = 374.52311 \\
 \text{Proof} = 376.00000 = \text{Summ}
 \end{array}
 \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Add}$$

The Relation the Rule for Subtraction of Decimals has to that of Vulgar Fractions.

In Vulgar Fractions the Rule to Subtract one from another, is first to Reduce them to one and the same Denominator, and then the difference of the Numerators placed over the Common Denominator is the Remainder or Answer.

And the very same is to be observed in Subtraction of Decimals: they needing no Reduction to a Common Denominator (as was said in Addition) being so already, so that the difference between the Numerators, which are the Decimals given, is the Answer; which shews that the nature of the operations in Decimals and Vulgar Fractions are the same; though the Work of Decimals be less troublesome, for the reason mentioned in this part, in Addition foregoing.

The

The Demonstration of Subtraction of Fractions.

As Fractions cannot be added (for the reasons foregoing in Addition) unless they are reduced to a common Denominator, so neither can the difference be discovered without that: For if the difference between 2 Fractions of different Denominators were required, as of $\frac{1}{2}$ and $\frac{1}{3}$, the Difference of the Numerators is 1, but that 1 is neither nor $\frac{1}{2}$, so that neither of those Denominators will serve, there must therefore be such a Denominator found as will serve both Fractions, which cannot be done unless the Numerators are altered, and made proportionable to that new or common Denominator.

As here, If you Multiply 3 (one of the Denominators given by 4, it will produce 12, and the other Denominator 4 multiplied by 3 is also 12, now if you Multiply the respective Numerators by the same Number you multiplied its Denominator, it will produce a new Numerator, which will have such proportion to the new Denominator, as the Primitive Numerator had to the Primitive Denominator.

Thus because I multiplied the Denominator 3 given by 4, I Multiply its Numerator 2 by the same Number, and the product is 8, so the new Fraction is $\frac{8}{12}$ equal to $\frac{2}{3}$ given; so likewise the other Denominator 4, multiplied by 3, and its Numerator by the same Number produces $\frac{3}{12}$, equal to $\frac{1}{4}$.

Now the Fractions being thus in a common Denominator as $\frac{8}{12}$ and $\frac{3}{12}$ the Difference is undeniably $\frac{5}{12}$; because in each of these new Fractions the Unit is divided into one and the same number of parts, and the proportion of 9 the new Numerator is to 12 its Denominator; as the Numerator 3 in the Fraction given is in proportion to its Denominator 4: And likewise 8 is to 12 in the other new Fraction, as 2 is to 3 in the other Fraction given, and consequently being in the same proportion the Fractions must be of the same value with those given, because the value of Fractions consists in the difference of the Numerator and Denominator.

So the difference between 9 and 12 in the new Fraction is $\frac{3}{12}$, which is equal to $\frac{1}{4}$, the difference between the 3 and 4 in the Fraction given: so that $\frac{1}{2}$ and $\frac{1}{4}$ is of the same value because $\frac{1}{2}$ makes each of them a Unit, and so of any other Fraction.

The reason why Decimals are subtracted as whole Numbers you have next before the use of Decimals.

CHAP.

C H A P. V.

*Multiplication of Decimals.**Rule.*

Multiply your Decimals in every respect, as though they were whole Numbers, and cut off for Decimals so many places toward the Right hand of the product, as there are Decimal places both in the Multiplicand and Multiplier.

And if so many places be not found in the product, you must make up the Number, by placing Cyphers toward the Left hand next the point.

Example 1.

Multiply by	$\begin{array}{r} 34.25 \\ 3.2 \\ \hline 6850 \\ 10275 \\ \hline 109.600 \end{array}$	} A mixt Number by a mixt.
	109.600 product	

To Multiply Shillings and Pence by Shillings and Pence, &c. by Decimals, &c.

Under this Example of Multiplying a mixt number by a mixt will fall the Solution of those pretended Problems, proposed by some who fancy them sufficient Tests to try the abilities of an Accomptant, though there is nothing more easie to those that understand either Vulgar Fractions or Decimals: the Questions are of this Nature; say they.

What is the product of 3 s. 3 d. by 6 s. 6 d.

The

The Answer by Decimals is thus :

s.	d.		s.	
3	3	Equal to	3.25	} Multiply
6	6	is Equal to	6.5	

$$\begin{array}{r} 1625 \\ 1950 \\ \hline \end{array}$$

21.125 Shillings for Answer.

The value of the Decimal in the product .125 of a Shilling, is $1\frac{1}{2}d$ so that the Answer is 21 s. $1\frac{1}{2}d$.

This and the like Questions are performed by Vulgar Fractions *By Vulgar Fractions.*

$3\frac{3}{12} s.$ by $6\frac{6}{12}$.

These Numbers in Fractions are $\frac{39}{12}$ by $\frac{78}{12}$ and their product (as by the Rules given for Vulgar Fractions is $\frac{3042}{144} s.$ or 21 s. $1\frac{1}{2}d$. as before.

But some are apt to take the product of 39 d. by 78 d. which is the Numerator, 3042 for the Answer in Pence, and think the Rules of Reduction are not right, because that product is not the true Answer.

I am sure if that were the Answer, the Rules for Multiplication both of Vulgar Fractions and Decimals must be wrong; which cannot be, as is easily shown by undeniable Reason and Demonstrations; (some of which follows) but that it is not applicable to any Rule in Reduction to take the 3042 for the answer in pence I am very well assured, nor is there any foundation for such an opinion either from Rule or Reason, 3042 pence being just 12-times the true answer 21 s. $1\frac{1}{2}d$. because they divide the 3042 by 12 instead of 144, as appears plain from the method of working the Question by Vulgar Fractions above.

Example 2.

Multiplication of Decimals.

Example 2.

$$\begin{array}{r}
 \text{Multiply} \quad 34.25 \\
 \text{by} \quad .32 \\
 \hline
 6850 \\
 10275 \\
 \hline
 10.9600 \text{ Product.}
 \end{array}
 \quad \left. \vphantom{\begin{array}{r} 34.25 \\ .32 \end{array}} \right\} \text{A mixt Number by a Decimal.}$$

Example 3.

$$\begin{array}{r}
 \text{Multiply} \quad 34.25 \\
 \text{by} \quad 32 \\
 \hline
 6850 \\
 10275 \\
 \hline
 1096.00 \text{ Product.}
 \end{array}
 \quad \left. \vphantom{\begin{array}{r} 34.25 \\ 32 \end{array}} \right\} \text{Mixt Numb. by a whole Numb.}$$

Example 4.

$$\begin{array}{r}
 \text{Multiply} \quad .3425 \\
 \text{by} \quad 32 \\
 \hline
 6850 \\
 10275 \\
 \hline
 10.9600 \text{ Products.}
 \end{array}
 \quad \left. \vphantom{\begin{array}{r} .3425 \\ 32 \end{array}} \right\} \text{A Decimal by a whole Number.}$$

Example 5.

Example 5.

$$\begin{array}{r}
 .3425 \\
 .32 \\
 \hline
 6850 \\
 10275 \\
 \hline
 .109600 \text{ Product.}
 \end{array}
 \left. \vphantom{\begin{array}{r} .3425 \\ .32 \end{array}} \right\} \text{A Decimal by a Decimal.}$$

A second Example of a Decimal by a Decimal.

$$\begin{array}{r}
 .1723 \\
 .012 \\
 \hline
 3446 \\
 1723 \\
 \hline
 .0020676 \text{ Product.}
 \end{array}$$

The Relation that Multiplication of Decimals has to that of Vulgar Fractions.

The Rule to Multiply Vulgar Fractions together, is to Multiply the Numerators together for a new Numerator, and the Denominators together for a new Denominator.

And in the very same manner we Multiply Decimals, to instance in the last Example, $\frac{1723}{10000}$ by $\frac{12}{1000}$ the product of the Numerators 1723, and 12 is 20676; and the product of the Denominator 10000 by the other 1000 produceth 10000000; so that the new Numerator 20676 being placed over the new Denominator 10000000, the Fraction which is the product is $\frac{20676}{10000000}$, which expressed without its Denominator, as by the 4th step, 2d. General head, of Chap. 1st. is .0020676 = the product above.

E

The

The Demonstration of Multiplication of Fractions.

It may seem strange to some that the product is not greater than the Multiplier, or Multiplicand, since it is always greater in whole Numbers.

Why the Product in Multiplication of Fractions is less than either of the Factors.

But the Reason is plain, since the multiplying Units encreases those Units *ad infinitum*; but the Multiplication of parts of a Unit does still decrease, or lessen the parts to infinite littleness; so that if a Fraction be multiplied by 1, it produceth the Fraction given, as $\frac{1}{10}$ multiplied by 1 produceth $\frac{1}{10}$; but if that $\frac{1}{10}$ be multiplied by less than 1 (*i. e.* by any Fraction) then it follows that the product must be proportionably less than $\frac{1}{10}$, as the Multiplier is proportionably less than 1: Thus if $\frac{1}{10}$ be multiplied by $\frac{1}{10}$, or half of 1, the product will be as much less than $\frac{1}{10}$ (the Fraction given) as the Multiplier $\frac{1}{10}$ is less than a Unit: as $\frac{1}{10}$ by $\frac{1}{10}$ produces $\frac{1}{100}$ (or $\frac{1}{4}$) which is $\frac{1}{2}$ the Multiplier, because that Multiplier is but half of 1.

The Multiplicand and Multiplier are a Compound Fraction of Unity.

Hence it is evident, That to Multiply any Number by a Fraction, is nothing but to take such a part of the Multiplicand for the product, as the Multiplier is of 1: As to Multiply 24 (or any other Number) by $\frac{1}{10}$ (or $\frac{1}{2}$) is to take $\frac{1}{10}$ or $\frac{1}{2}$ of 24, which is 12 for the product, and by the same Rule $\frac{1}{2}$ multiplied by $\frac{1}{4}$ produces a quarter of $\frac{1}{2}$, which is $\frac{1}{8}$, equal to $\frac{1}{4}$ of $\frac{1}{2}$ of 1.

Hence it is also evident, That to Multiply $\frac{1}{2}$ by $\frac{1}{4}$ being only to take $\frac{1}{4}$ of $\frac{1}{2}$ of a Unit: The Multiplicand and Multiplier are one Compound Fraction of a Unit.

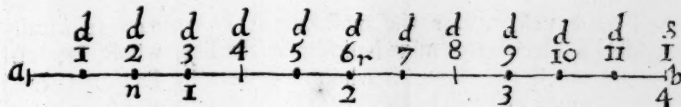
And to reduce any Compound Fraction to a simple one of the same value, is to Multiply the Numerators together for a new Numerator, and also the Denominators for a new Denominator, according to the Rule in the relation last foregoing.

The reason of reducing Compound to simple Fractions.

Now, that such Multiplication will reduce Compound Fractions to simple ones of the same value, appears best by an instance of what is undeniable and plain, for Example, It is plain and beyond dispute, that $\frac{2}{3}$ of $\frac{3}{4}$ of 1 Shilling Sterling is 6 pence; For the Numerators 2 and 3 multiplied together is 6, and 3 by 4 the Denominators is 12, so $\frac{6}{12}$ is the product, now $\frac{1}{12}$ of a Shilling being 1 Penny, and consequently $\frac{6}{12}$ six pence proves the Rule to be true.

Or

Or to make this yet more apparent, The line *ab* is one Shilling divided into 12 equal parts or pence.



Likewise the same line divided into 4 equal parts, and each of those 4 into 3 equal parts; then will $\frac{1}{3}$ of $\frac{1}{4}$ be $\frac{1}{12}$ of the whole line (*ab*) for in 1 fourth there are 3 parts, and consequently in 4 fourths (or the whole line, or Unit) there must be 12 of these parts; which shews the Reason why the Denominators are to be multiplied together for the Denominator of a simple Fraction.

And the Reason why the Numerators are multiplied together (as in the Fraction above, $\frac{1}{3}$ of $\frac{3}{4}$) is because $\frac{2}{3}$ of $\frac{1}{4}$ of the line (*ab*) as (*an*) is but $\frac{1}{3}$ part of the Fraction $\frac{2}{3}$ of $\frac{3}{4}$ (as the line (*ar*) therefore $\frac{2}{3}$ of $\frac{3}{4}$ or (*ar*) or $\frac{6}{12}$ is 3 times (*a2*) or (*an*) which shews why the Numerator 2 is to be multiplied by 3, to give the Number of the twelfth parts in the simple Fraction of the line (*ab*) required.

The reason why Multiplication of Decimals is performed as whole Numbers, is shewn next before the use of Decimals.

C H A P. VI.

Division of Decimals.

Division by Decimals is performed in every respect as that of whole Numbers. So that there's no difficulty in it; and after Division is made, take the following Rule for giving the true value of the Quotient.

A General Rule for the Quotient.

For knowing what places in the Quotient are Decimal, take the difference between the Decimal places in the Dividend, and those in the Divisor, and so many as that difference is, you must separate

The Reason of this, is shewn in the Demonstration of Division.

by a point from the Quotient toward the Right hand thereof, and if there are not so many, make up the Number by Cyphers toward the left hand.

And forasmuch, as for the most part it is necessary (especially in dividing a Decimal or mixt Number by the like, whose Decimal places are much the same, or whole Numbers by Decimals, &c.) To add Cyphers to the Dividend, that so a Competent Number of Decimal places may come out in the Quotient; therefore to know in all cases what Cyphers to add to the Dividend in places toward the Right hand, take this Rule.

A General Rule for the Dividend.

The Reason of this is shewn in the Demonstration of Division of Decimals. To the Number of Decimal places in the Divisor, add the Number of Decimal places that you would have in the Quotient (as 3 is sufficient if the Quotient is not to be multiplied by any thing, and there happen to be a remainder in dividing) and as many as that Summ is, make so many Decimal places in the Dividend, by adding Cyphers if requisite.

Example 1.

To divide a mixt Number by a mixt, as 109.6 by 3.2?

$$\begin{array}{r}
 3.2 \overline{) 109.6000} \quad (34.250 \\
 \underline{96} \\
 136 \\
 \underline{128} \\
 80 \\
 \underline{64} \\
 160 \\
 \underline{160} \\
 00
 \end{array}$$

Note,

Note, This proves the first Example in Multiplication of Decimals.

Example 2.

To divide a mixt Number by a Decimal, as 10.96 by .32.

$$\begin{array}{r}
 .32 \overline{) 10.9600} \quad (34.25 \\
 \underline{96} \\
 136 \\
 \underline{128} \\
 80 \\
 \underline{64} \\
 160 \\
 \underline{160} \\
 0
 \end{array}$$

Note that this proves the second Example in Multiplication of Decimals; and though I add 3 Cyphers, that so I may have 3 Decimal places in the Quotient according to the last General Rule; yet because nothing remains before I have divided the Cypher in Units place, I therefore Cancel it as useless, and according to the first General Rule in Division, I say the difference between 4 Decimal places in the Dividend, and 2 in the Divisor is 2, therefore I separate 2 of the first figures toward the Right hand the Quotient for Decimals.

Example 3.

Example 3.

To divide a whole Number by a whole Number, so as to have a Competent Number of Decimals in the Quotient, as 1096 by 32.

$$\begin{array}{r}
 32 \overline{) 1096.000} \quad (.3425) \\
 \underline{136} \\
 128 \\
 \underline{00} \\
 80 \\
 \underline{64} \\
 160 \\
 \underline{160} \\
 0
 \end{array}$$

Note, This proves the truth of the third Example in Multiplication.

Example 4.

To divide a mixt Number by a whole Number that is greater than the Dividend, as 10.96 by 32.

$$\begin{array}{r}
 32 \overline{) 10.9600} \quad (.3425) \\
 \underline{136} \\
 128 \\
 \underline{00} \\
 80 \\
 \underline{64} \\
 160 \\
 \underline{160} \\
 0
 \end{array}$$

Example 5.

Example 5.

To divide a Decimal by a Decimal, as .0020676 by .1723.

$$\begin{array}{r}
 .1723 \overline{) .0020676} \quad (.012 \\
 \underline{1723} \\
 3446 \\
 \underline{3446} \\
 0
 \end{array}$$

Note, This proves the truth of the last Example in Multiplication of Decimals.

Example 6.

To divide a Decimal by a whole Number, as .9 by 324.

$$\begin{array}{r}
 324 \overline{) .9000} \quad (.0027 \\
 \underline{648} \\
 2520 \\
 \underline{2268} \\
 252 \text{ Remains.}
 \end{array}$$

Note, That in the two last Examples, the difference between the Decimal places in the Dividend and Divisor, is more than there are places in the Quotient: Therefore according to the first Rule, I prefix Cyphers toward the Left hand in the Quotient to make up such Number.

The Relation that Division of Decimals has to that of Vulgar Fractions.

There is no Rule in Decimals performed more contrary to Vulgar Fractions than Division, not but that Division of Decimals may be done the same way with that of Vulgar Fractions, or Division of Vulgar

Decimals may be Divided as Vulgar Fractions and the contrary; as by the Examples following.

Rule.

Fractions the same way that Decimals are ; but then in Decimals there will always be a superfluous number of Cyphers, both in the Numerator and Denominator, if worked as Vulgar Fractions, the Fractions (when Division is performed *By multiplying the Numerator of the Divisor into the Denominator of the Dividend for the Denominator of the Quotient, and the Denominator of the Divisor in the Numerator of the Dividend for the Numerator of the Quotient*) being so far from their lowest Terms, that their values are seldom or never perceivable without reducing.

Why Decimals must not be divided as Vulgar Fractions.

For instance, Suppose $\frac{5}{100}$ is to be divided by $\frac{25}{100}$ in Vulgar Fraction ways, according to that Rule, the Quotient is $\frac{100}{2500}$, so that in this Vulgar Fraction way of Dividing Decimals, here is not only 2

$$\frac{25}{100} \bigg) \frac{5}{10} \left(\frac{500}{250} \right)$$

Cyphers, one in the Numerator and the other in the Denominator superfluous ; but when they are cut off the Quotient is $\frac{10}{25}$, which before you can know the value of, you must divide 50 by 25, according to the Rules foregoing, which was all that was required at first, if to be divided Decimal way ; To that all that has been done thus far to dividing these Decimals as Vulgar Fractions is found unnecessary.

2. Therefore the shortest and best way of dividing Decimals is to divide only the Numerator of the Dividend by that of the Divisor, adding a Cypher or Cyphers (when too little to be divided, as in the last Example above) both to the Numerator and Denominator.

3. And also divide the Denominator of the Dividend by that of the Divisor, and the Quotients will be the respective Numerators and Denominators of the Quotient required.

Thus $\frac{5}{100}$ divided by $\frac{25}{100}$ (by adding 1 Cypher to 5, and another to 10, making the Numerator of the Dividend 50, and the Denominator 100) the Quotient will be $\frac{2}{5}$ or 2 ; but more of this in the Demonstration following.

4. And by the same Rule may Division of Vulgar Fractions be performed (though not so good a way) as in the Vulgar Fractions, agreeing with the foresaid Decimals, to divide

$\frac{1}{2}$ by $\frac{1}{4}$, because 2 the Denominator of the Dividend cannot be divided by 4, the Denominator of the Divisor, therefore I add a Cypher to 2, and 1 in the Dividend making it $\frac{10}{20}$, and then the 20 divided by 4 quotes 5, and the 10 by 1 quotes 10 : So is the Quotient $\frac{10}{5}$ or 2 as before.

$$\frac{1}{4} \bigg) \frac{10}{20} \left(\frac{10}{5} \text{ or } 2 \right)$$

5. But.

5. But because the Numerator and Denominator of the Dividend cannot always be divided by those of the Divisor without Remainders not being certain as Decimals; it is therefore more exact, and is used in dividing Vulgar Fractions, to Multiply the Numerators and Denominators of the Dividend and Divisor cross, as by the Rule foregoing; but more of this in the Denominator following.

The demonstration of Division of Fractions.

In demonstrating this, I shall shew first the reason of the Rule for Vulgar Fractions, and then for that of Division of Decimals.

1. *To augment the value of any Fraction is nothing but to Multiply, or add to its Numerator, or to take from, or divide the Denominator;* Because a Fraction is nearer or farther from a Unit, as the Numerator is nearer or farther from the value of the Denominator; for $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, &c. are each a Unit; but then in comparing 2 Fractions, respect must be had to the Denominators; for the less that is (when there is the same difference between the Numerators and Denominators) the greater is the Fraction thus: $\frac{2}{3}$ is more than $\frac{7}{11}$, though there be the same difference between the Numerators and Denominators, because $\frac{2}{3}$ wants but $\frac{1}{3}$ of a Unit, whereas $\frac{7}{11}$ wants $\frac{4}{11}$ of a Unit; now $\frac{1}{11}$ of any thing being more than $\frac{1}{13}$, therefore that which wants $\frac{4}{11}$ of being a Unit, wants more than that which wants but $\frac{4}{13}$ of being so. And hence it follows

2. *That to make a Fraction less is to encrease or add to the Denominator.*

For as the Fraction (for Example) $\frac{3}{4}$ of a yard is made twice so much if the Numerator 3 is multiplied by 2, viz. $\frac{6}{4}$, or $1\frac{1}{2}$ yard; so is that same $\frac{3}{4}$ made but half so much, viz. $\frac{3}{8}$ (or 1 quarter and $\frac{1}{2}$ Quarter of a yard) if the Denominator 4 be multiplied by 2.

3. Now from these 2 Actions thus made out, the reason of the Rule for dividing Vulgar Fractions will appear. For admit $\frac{1}{4}$ is to be divided by $\frac{1}{4}$, if it were divided by a Unit the Quotient would be $\frac{4}{4}$; and therefore since it is to be divided but by $\frac{1}{4}$ of 1, the Quotient must be 4 times as much as $\frac{4}{4}$.

F

4. For

The Reason of Division. 4. For as in whole Numbers, so in Fractions, by how much the less the value of the Divisor is by so much the more will the Quotient be; which in our Example, (according to the first Axiom) is $\frac{16}{5}$ or 4 times $\frac{4}{5}$, which is a plain reason why the

Numerator of the Dividend is multiplied by the Denominator of the Divisor to give the Numerator of the Quotient.

$$\frac{1}{4}) \frac{4}{5} \left(\frac{16}{5}$$

5. Now the reason why the Denominator of the dividend is to be multiplied by the Numerator of the Divisor is this: Suppose the $\frac{4}{5}$ were to be divided by $\frac{3}{4}$; by the 4th step, the Quotient will be but $\frac{1}{3}$ part of $\frac{16}{5}$ because $\frac{3}{4}$ (this new Divisor) is 3 times as much as $\frac{1}{4}$ (the former Divisor) therefore to make the Quotient $\frac{16}{5}$, to be $\frac{1}{3}$ less, is (according to the second Axiom) to Multiply its Denominator by 3 the Numerator in this second Example, and the product is the Denominator of the Quotient sought.

$$\frac{3}{4}) \frac{4}{5} \left(\frac{16}{15}$$

6. So that in short by multiplying the Numerator of the dividend by the Denominator of the Divisor (as per the third and fourth step we make the Quotient proportionably greater than the dividend as the Divisor (taking the Numerator for 1) is less than Unity: and by multiplying the Denominator of the Dividend by the Numerator of the Divisor, we again make the Quotient proportionably less as the said Numerator is greater than 1, as per the first and second and fifth steps.

The Demonstration of Division of Decimal Fractions.

7. The reason of the Rule for the dividing of one Decimal by another is this: When the Decimal places in the Numerator of the Dividend are less or equal to those of the Divisor there is Cyphers added to make the Division more exact as in dividing .4 by .25, there can no error proceed from placing Cyphers to the Right hand the 4 that so 25 may be had therein, because (as is said before) A Cypher toward the Right hand in a Decimal neither encreaseth nor diminisheth the value thereof, $\frac{4}{10}$ the dividend given, being as much as .40, viz. $\frac{40}{100}$ or $\frac{400}{1000}$ because though 'tis true the Numerator is encreased by adding the Cyphers, yet the Denominator encreasing at the same time as much, makes the Decimal to retain the same value.

8. Now the most rational way of dividing one Fraction by another (as is declared in the relation of Division foregoing) is to divide the Numerator of the Dividend by that of the Divisor for the Numerator of the Quotient, and likewise the Denominators for that of the Quotient,

Quotient, so in the Example above, in dividing $\frac{1}{100}$ by $\frac{1}{25}$, I make $\frac{1}{100}$ into $\frac{400}{1000}$ that so I may have the Divisor 25 the other therein, whereby the Quotient will be more exact than if I should only divide 40 by the 25, so that 400 divided by 25 gives 16 for the Quotient, and the Denominator 1000 divided by the other which is 100, the Quotient is 10; so it appears that the Quotient is $\frac{16}{10}$, or 1.6. For this is the ground of that Rule for pricking off Decimals from the Quotient, which says the Number so separated must be equal to the difference between the Decimal places in the Dividend and Divisor, which is evident; because that difference is always equal to the Number of Cyphers in the Denominator of the Quotient; by which Denominator, if the Numerator of the Quotient be divided, that Quotient will always shew the true value of the Fraction, which Division is always performed without working, because any Number is divided by 10, 100, 1000, &c. by cutting off as many places in the Dividend, as there are Cyphers in the Divisor; so is 16 divided by 10, by cutting off 6, and the Quotient is 1.6.

And where the Quotient is of little value as in case of dividing a Decimal by a whole Number as in the Margent, the reason is the same, the Rules in Numeration of Decimals being observed for reading a Decimal Fraction without its Numerator, which will shew the reason of putting Cyphers toward the Left hand in such a Quotient, as here the Quotient $\frac{198}{100000}$ is expressed, without the Denominator 100000, by making the point to possess the hundred thousands place.

A second Example.

$$\frac{146}{1} \div \frac{29000}{100000} = \left(\frac{198}{100000} \right) \text{ or } .00198$$

The reason why Decimals are divided as whole Numbers is shewn in the next Chapter but one.

C H A P. VII.

The Rule of Proportion by Decimals.

THIS Rule being only Multiplication and Division, which are taught already; I might upon that account pass it by, but because Proportions are used in the Calculation of Interest, (which will be treated on afterwards) I shall give some Examples for the satisfaction of some, who may want the assistance of the same.

A Rule for stating your Question.

There being always 3 Numbers given to find a fourth: 2 of which are of (or are reducible to) one Denomination; therefore place that of a contrary Denomination to the said 2, down first; then to the Left hand that (or in the first place in course of Writing) place that Number of the said 2 that are of one name, which has dependance on that already placed down, and then place the third Number next the Left hand, which being done, for performing the operation, this is the Rule.

A Rule for working any Question.

This is the same with that in whole Numbers, for if you multiply the second and third Numbers together, and divide the product by the first Number, the Quotient is the fourth proportional Number.

Example 1.

If $1\frac{1}{2}$ Yards of Cloath cost 1 Pound 3 Shillings, what will a Piece of $24\frac{1}{2}$ Yards cost at that rate.

The parts of a Yard reduced into the Decimals of a Yard, and the Shillings into the Decimal of a Pound, as in the 9, 10, 11, 12, 13, 14 and 15 Rules in Reduction foregoing, the Numbers when stated will stand thus:

Yards

The Rule of Proportion by Decimals. 39

Yards *l.*
125 . 1.15 ::

Yards
245
1.15

1225

245

245

125) 28.8750 (22.54 Answer or *l.* 22
250 ... 10 s. 9½ d.

317

250

675

625

500

500

0

Example 2.

C. 2 lb.

If 12 : 3 : 25 of Sugar cost 75 *l.* 17 s. 6½ d. what will 112 Pound cost at that rate?

Q. *l.*

C.

The Decimal of a Hundred that 3 : 25 is, is .9732, and the 17 s. 6½ in the Decimal of a Pound is .877 therefore the work by the Rule is stated and worked thus :

CC

$$\begin{array}{r}
 \text{C.} \quad \text{L.} \quad \text{C.} \\
 12.9732. \quad 75.877. \quad 1 \\
 \hline
 12.9732) 75.8770000 (5.848 \text{ Answer, or } 1.5.16: \\
 \hline
 1101100 \\
 \hline
 632440 \\
 \hline
 1135120
 \end{array}$$

Another
way of
Division.

Note, That in this way of dividing, I have omitted putting down the products, having deducted the same as I multiplied, thus (in the first Figure 5) I say, 5 times 2 in the Divisor is 10, from 10 in the Dividend rests 0, and 5 times 3 is 15, and 1 carried is 16, from 17 rests 1, and 5 times 7 is 35, and 1 carried is 36, from 37 rests 1, and carry 3, 5 times 9 is 45, and 3 is 48, from 48 in the Dividend (borrowing 4) and the rest is 0, and 5 times 12 is 60, and 4 is 64, from 75 rests 11, which you see I put down, and the remainder is 110110, and so of the rest of the work.

The Demonstration of this Rule of Proportion.

The reason of Multiplication, and Division of Decimals have been made plain before; it only here remains to prove that Multiplying and Dividing, as *per* the last Rule above, will give a fourth proportional Number, that is, one that bears such proportion to the third, as the first does to the second; for whatever Number does so, is the true Number sought.

1. Admit .3 is in proportion to 2.7, as 12 is to a fourth Number unknown; for which fourth Number I put (*n*) then the proportion stands thus :

$$.3. \quad 2.7 :: 12. \quad n$$

2. And

Decimals wrought as whole Numbers. 41

2. And by the 16 of the 6 Euclid the Product of 3 into (u) the first and fourth is equal to the Product of 2.7 and 12 the 2 and 3 Numbers, thus:

$$.3 \ u \text{ is equal to } 32.4$$

3. Now it follows, that if the Product of u by .3 is equal to 32.4, then (u) is equal to 32.4 divided by .3 thus:

$$u \text{ equal to } \frac{32.4}{.3}$$

Now 32.4 divided by .3 gives the fourth proportional Number, viz. 108, which is the Answer to the Question. And if the second, third and fourth steps are compared, you will find that this Number 108 is discovered in the very same way, as *per the Rule for working any Question*, foregoing.

C H A P. VIII.

The Reason why the operations in the Rules foregoing by Decimals, are performed as in those of whole Numbers.

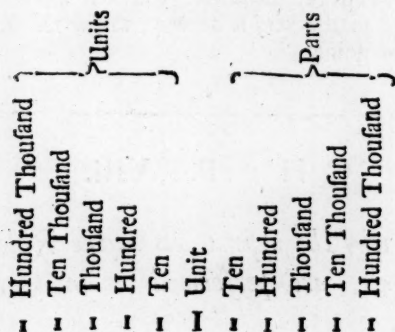
HAVING in the Chapters before shewed the Nature of Fractions in General, and the Relation Decimals have to Vulgar Fractions, both in the Nature of the Fractions themselves and in their operations, I shall here shew the Relation Decimals have to whole Numbers, and why they are wrought alike.

That Decimals should be wrought as whole Numbers, seems very unaccountable to some, who have not well considered the matter: I shall therefore give the Reason why they are so: which proceeds from the Degrees of Figures places, and the Nature of the Denominators of Decimals.

Id:

In Decimal Fractions the Unit being (as has been sufficiently explained) divided into 10 parts, and each 10th part into a 10, each 100th part into 10, &c. makes the Subdivision of the parts to have the same decrease or increase that whole Numbers have; which is the chief Reason why the Adding, Subtracting, Multiplying, Dividing, &c. of them agrees exactly with whole Numbers. For instance,

There is the same Geometrical proportion between 1 tenth part of a Unit and a Unit, as there is between a Unit and 10; for 10 Tenth parts is 1, and Ten Units is 10; so that if you suppose a Unit divided into 100000 parts the encrease of every place from the Right hand toward the Left, is by 10 not only till you come to a Unit; but until you come to 100000 Units, &c. *ad infinitum*.



Some Authors have so confounded Fractions and whole Numbers with the useless Terms of decreasing and encreasing Numbers, as though they were quite different in their Numeration: whereas there is nothing more plain than that the value of each of these Numbers above Unity, does gradually decrease (according to the places) to the 1 hundred thousandth part of a Unit; and on the contrary the value of each Number encreaseth gradually, according to the place from one hundred thousand part of a Unit, or lower to 100000, &c. Units or higher; so that there is no manner of difference in the encrease or decrease of decimal Parts and Units, they making together as it were one infinite line, of which a Unit is the Center.

And

And every ten Decimal parts in one and the same place let the place be were it will, making one of the parts in the next place towards the Left hand, just as 10 in any place of whole Numbers makes one in the next place toward the Left hand: It must needs follow that there being no difference between the increase or decrease of Decimal places and those of whole Numbers, there can be none in the manner of Adding, Subtracting, &c. of Decimals and those of whole Numbers.

2. As to the separation of places for Decimals from the Totals, Remainders, Products, and Quotients after the work of Addition, Subtraction, Multiplication, or Division is ended, the Reason is purely from the Number of Cyphers that the Denominators would consist of if wrote down; which being always a Unit with Cyphers towards the Right hand, divides any Number only by a point of separation of so many places, as the said Denominator has Cyphers in it: But this having been so fully discussed in the Relations and Demonstrations foregoing, I need to say no more thereof in this place.

CHAP. IX.

*Containing the Excellent use of Decimals in General,
and that they exceed any other Fraction.*

1. **T**HE Excellency of Decimals does sufficiently appear (if there were nothing else to be said for them) from their ^{Decimal} Nature being like whole Numbers, as in the last Chap- ^{Fractions} ter has been Explained, upon which account it is im- ^{the best.} possible there should ever be any kind of Fraction invented so ex- ^{1. Because} cellent as Decimal; because no other Fraction can be expressed with- ^{wrought as} out the Denominators, and wrought as though they were Integers; ^{whole Num-} these alone retaining the same proportion between their degrees or ^{bers.} places that whole Numbers do, and consequently none can be wrought as whole Numbers but these, And

G

2. In

2. *From the Nature of the Denominator. How made preferable to Duodecimals in Measuring.*

2. In almost all operations by Fractions there is occasion to divide some Number by the Denominator, which in Decimals only is such as may be performed without work (as I have shewed above) I know there are some that put a great value upon Duodecimals, in the Mensuration of all manner of Work, as Wainscor, Painting, Plastering, &c. but let but the Rule wherewith the Dimentions are taken, viz. the Yard, Foot, &c. be divided into 100 parts, according to the Nature of a Decimal Fraction, and the work is done by an easie Multiplication of one Denomination, whereas in other ways of Measuring work, there are many tedious Reductions and Divisions, which are avoided by the Decimal way.

3. *In dividing the Lines on Mathematical Instruments*

3. Another thing wherein the Excellent use of Decimals appears, is their aptness for dividing any Measure into parts, which parts agree, so exactly with those of whole Numbers, that the parts serve either as Units themselves, or as parts of a Unit, as the Divisions on Diagonal and other Scales, and Instruments of singular use in all the parts of the Mathematick; all or most of which are Decimally divided.

4. *In the precise and easie Calculation of Tables.*

4. The Excellency of Decimal Fractions above all others that can possibly be thought on, appears in their usefulness in the precise and speedy Calculation of all manner of Tables; and more especially in those two excellent ones, of Compound Interest and of Logarithms; which if they were to be calculated by any other Fractions but Decimals, would not only be four or five times as long in perfecting; but when done, would be abundantly more troublesome in the using of those Numbers when Calculated, in all which, besides in things of lesser moment, I have very much experienced the exceeding usefulness of Decimals.

5. *In computing Interest especially Compound.*

5. Decimals are of admirable use in the calculating the Interest due for any Summ of Money, which may be found of 5 Summs of Money, by Decimals sooner than that of 1 Summ by Reduction; which I shall therefore shew more particularly in

CHAP. X.

Concerning Interest of Money.

IN Discourſing on this Subject, I ſhall ſhew you

1. What Principal Money is.
2. What Interest is both Simple and Compound.
3. What the Rate of Interest is.
4. What Discount of Money is.
5. How to find the Simple Interest of any Summ.
6. To find the Compound Interest of any Summ.
7. To find the true Discount of any Summ for any time.
8. Give you 2 Tables, the one of Simple, the other of Compound Interest.
9. Shew the uſe of the Table of Simple Interest.
10. Of that of Compound Interest.
11. How to Calculate any of the Numbers in the Compound Interest Table.

1. Principal Money is the Money that is put out to Interest, *Principal* which is probably called ſo; becauſe it is always the Stock or Summ *what.* which begets or produces Interest.

2. So that Interest Money, or Uſury is the Summ paid to the *Interest* Lender of the Principal by the Borrower, for his Interest therein; *Simple, and* which is Simple or ſingle Interest, when the Interest of the Summ *Compound* actually received by the Borrower is only paid. *what.*

But Compound Interest, is the Interest not only of the Original Principal Lent; but alſo of ſuch Interest as is not paid when due, being added to the Principal firſt Lent; as if I have 100*l.* at Interest for a Year, for which I am to receive 6*l.* but that not being paid when due, I reckon Compound Interest for the ſecond Year, *i. e.* the Interest of the 1.100, and of the 6 Pounds that ſhould have been paid for Interest the firſt Year.

3. The rate of Interest is the Summ that is paid for Interest of 1.100 for 1 Year; for it is from that Rate, Reason or Proportion *Rate of Interest what.* that the Interest of any other Summ for any other time is Computed.

G 2

4. The

Concerning Interest of Money.

*Discount,
what it is.*

4. The Discount of Money, is the abating of part of the Principal for paying the Remainder before it would grow due; as if I have *l.* 100 due to me at the end of 1 Year; but having present occasion for Money, I discount at the rate of 6 *per Cent. per Ann.* so that I must receive but *l.* 94 : 6 : 9½ presently, to satisfy my Debt of *l.* 100 due to me a Year hence. How this is done will appear in the 7th. head of this Chapter.

*To find the
Simple In-
terest of
any Summ
for 1 Year.*

5. The way to find the Simple Interest, or increase of any Summ for any time; as suppose of 520 for 1 Year, is thus : say As *l.* 100, is to 100, and the Interest thereof for a Year, so is 520 (or any other Summ) to the Summ it is amounted, or encreased unto in that time.

1. Rule.

See the Work.

<i>Prin.</i>	<i>P. and Int.</i>	<i>Prin.</i>
100.	106 ::	520.
		106
		<hr/>
		312
		52
		<hr/>

Answer = *l.* 551.20

*For any
Number of
Years.*

2. Rule.

Here you see that the *l.* 520 is encreased to 551 : 4 s. in 1 Year, at the Rate of 6 *per Cent. per Ann.* or if you would know the Simple Interest for any Number of Years, first Multiply the Rate by the Number of Years, and add the Summ to 100 *l.* then

As 100 is to that Summ Total,

So is the Principal given, to the amount required.

Example

Example 1.

What is the amount or encrease of *l.* 520 for 10 Years at 6 per Cent per Ann. Simple Interest?

According to the Rule 10 times the Rate is 60 *l.* which added to 100 makes 160, then

<i>Prin.</i>	<i>P. and Int.</i>	<i>Prin.</i>
As 100.	160 ::	520.
		160

312

52

l. 832.00 Answer

Here you see that *l.* 520 in 10 Years will encrease to *l.* 832 at the rate aforesaid; and the difference between *l.* 832 and *l.* 520, the Principal is *l.* 312, the Interest alone.

Again, If you would know the Simple Interest of 520 *l.* or other Summ for half or quarter of a Year: Take half or a quarter of the Rate, and add to 100 *l.* working then as before.

Interest for
4th parts
of a Year.
3. Rule.

Example 2.

What is the Interest of *l.* 520 for $\frac{1}{4}$ of a Year?

100.	101.5 ::	520
		520

2030

50.75

527.800

Here

Interest for
any Number
of Days.
4. Rule.

Here it is plain, that the amount of $l. 520$ for a quarter of a Year is $l. 527.8$ or the Interest is $l. 7: 16: 0$

Lastly, If you would find the Simple Interest of any Summ for any time, that is not the fourth part of a Year, Multiply $.000164383$ (which is the Interest of $1 l.$ for 1 day) by the days for which you would know the Interest, and that Product by the Summ given, and that last product is the Answer.

Example 3.

What is the Interest of $l. 120: 6: 0$ for 211 days?

$.000164383$ } Multiply
 211

164383
 164383
 328766

$.034684813$ } Multiply
 120.3

104054439
 69369626
 34684813

4.1725830039 Answer.

Here you see the Answer by the Rule is $l. 4: 3: 5\frac{1}{4}$

A Rule to
find the
Compound
Interest of
any Summ.

6. To find the Compound Interest of any Summ: You must have an operation in the Golden Rule for each of the Years the Summ is forborn; the first is to be wrought as in the 1 Rule of the last head, and the fourth Number in that must be the third Number in the second Operation; the fourth in the second must be the third in the third, &c. the 2 first Numbers in each operation being the same, as the amount of $l. 520: 4$ Years forborn is $l. 656: 9: 9$, thus:

$l. 100.$

	<i>l.</i>	<i>l.</i>	<i>l.</i>	<i>l.</i>
For the amount the first Year, as	100.	106::	520.	551.2
For the second Year,	100.	106::	551.2.	584.272
For the third Year,	100.	106::	584.272	619.32832
For the fourth Year,	100.	106::	619.32832	656.488

Here you see that if *l.* 100 require 106, *l.* 520 will require *l.* 551.2 at the end of 1 Year, and that will require *l.* 584.272 the second Year, and that *l.* 619.32832 the third, and that requires *l.* 656.488 the fourth Year.

As for the other things in Compound Interest, as the present worth of any Summ due any numbers of Years hence; the amount of Annuities, &c. I shall shew how to Calculate the same under the 11th. General head following.

7. To find the Discount to be allowed for paying any Summ before due; some only deduct the Interest, as if I have *l.* 520 due to me at the end of 1 Year; they usually deduct the Interest for that time, which by the first Rule to the 5th. General head of this Chapter is *l.* 31: 4: 00, and being deducted from the *l.* 520 leaves *l.* 488: 16: 00 to be paid presently: but the true method of finding the Discount upon *l.* 520, that is but due a Year hence to know what is payable presently, is by this Proportion. *The Discount how to find.*

As *l.* 106 is to 100: so is *l.* 520, to *l.* 490: 11: 33, which fourth number is the Summ to be paid presently.

So likewise the Discount of any other Summ, for any other time is found, by first finding the Interest of *l.* 100, for the time that the Money is paid before due: then

As the Summ of that Interest and *l.* 100, is in Proportion to *l.* 100: General Rule. so is any other Summ whatever to the Summ to be paid presently to satisfy the Debt, Discount being allowed of *l.* 6 per Cent. per Ann. for prompt payment.

8. The Tables of Simple and Compound Interest follow.

Simple

Simple Interest at 6 per Cent.

Princi- pal.	One Day,			Two Days,			Three Days,			Four Days,		
	l.	s.	d. q.	l.	s.	d. q.	l.	s.	d. q.	l.	s.	d. q.
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0
60	0	0	0	0	0	0	0	0	0	0	0	0
70	0	0	0	0	0	0	0	0	0	0	0	0
80	0	0	0	0	0	0	0	0	0	0	0	0
90	0	0	0	0	0	0	0	0	0	0	0	0
100	0	0	0	0	0	0	0	0	0	0	0	0
200	0	0	0	0	0	0	0	0	0	0	0	0
300	0	0	0	0	0	0	0	0	0	0	0	0
400	0	0	0	0	0	0	0	0	0	0	0	0
500	0	0	0	0	0	0	0	0	0	0	0	0
600	0	0	0	0	0	0	0	0	0	0	0	0
700	0	0	0	0	0	0	0	0	0	0	0	0
800	0	0	0	0	0	0	0	0	0	0	0	0
900	0	0	0	0	0	0	0	0	0	0	0	0
1000	0	0	0	0	0	0	0	0	0	0	0	0

A TABLE of Simple Interest at 6 per. Cent.

Principal.	Five Days.				Six Days.				Seven Days.				Eight Days.			
	l	s	d	q	l	s	d	q	l	s	d	q	l	s	d	q
s. 14	0	0	0		0	0	0		0	0	0		0	0	0	1
15	0	0	0		0	0	0		0	0	0		0	0	0	1
16	0	0	0		0	0	0		0	0	0	1	0	0	0	1
17	0	0	0		0	0	0		0	0	0	1	0	0	0	1
18	0	0	0		0	0	0	1	0	0	0	1	0	0	0	1
19	0	0	0		0	0	0	1	0	0	0	1	0	0	0	1
l. 1	0	0	0		0	0	0	1	0	0	0	1	0	0	0	1
2	0	0	0	1	0	0	0	2	0	0	0	2	0	0	0	2
3	0	0	0	2	0	0	0	3	0	0	0	3	0	0	0	1
4	0	0	0	3	0	0	0	1	0	0	0	1	0	0	0	1
5	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1
6	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1
7	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1
8	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1
9	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1
10	0	0	0	2	0	0	0	2	0	0	0	2	0	0	0	2
20	0	0	0	4	0	0	0	4	0	0	0	4	0	0	0	4
30	0	0	0	5	0	0	0	7	0	0	0	8	0	0	0	9
40	0	0	0	7	0	0	0	9	0	0	0	11	0	0	0	12
50	0	0	0	9	0	0	0	11	0	0	0	13	0	0	0	14
60	0	0	0	11	0	0	0	13	0	0	0	15	0	0	0	16
70	0	0	0	13	0	0	0	15	0	0	0	17	0	0	0	18
80	0	0	0	15	0	0	0	17	0	0	0	19	0	0	0	20
90	0	0	0	17	0	0	0	19	0	0	0	21	0	0	0	22
100	0	0	0	19	0	0	0	21	0	0	0	23	0	0	0	24
200	0	0	3	3	0	0	3	11	0	0	4	7	0	0	5	3
300	0	0	4	11	0	0	5	11	0	0	6	10	0	0	7	10
400	0	0	6	6	0	0	7	10	0	0	9	22	0	0	10	6
500	0	0	8	2	0	0	9	10	0	0	11	6	0	0	13	13
600	0	0	9	10	0	0	11	10	0	0	13	9	0	0	15	9
700	0	0	11	6	0	0	13	9	0	0	16	11	0	0	18	5
800	0	0	13	13	0	0	15	9	0	0	18	4	0	0	1	10
900	0	0	14	9	0	0	17	9	0	0	1	0	8	0	1	3
1000	0	0	16	5	0	0	19	8	0	0	1	3	0	0	1	6

A TABLE of Simple Interest at 6 per Cent.

Principal	9 Days,			10 Days,			11 Days,			12 Days,		
	l.	s.	d. q.	l.	s.	d. q.	l.	s.	d. q.	l.	s.	d. q.
s. 11	0	0	0	0	0	0	0	0	0	0	0	0 1
12	0	0	0	0	0	0 1	0	0	0 1	0	0	0 1
13	0	0	0 1	0	0	0 1	0	0	0 1	0	0	0 1
14	0	0	0 1	0	0	0 1	0	0	0 1	0	0	0 1
15	0	0	0 1	0	0	0 1	0	0	0 1	0	0	0 1
16	0	0	0 1	0	0	0 1	0	0	0 1	0	0	0 1
17	0	0	0 1	0	0	0 1	0	0	0 1	0	0	0 1
18	0	0	0 1	0	0	0 1	0	0	0 1	0	0	0 1
19	0	0	0 1	0	0	0 1	0	0	0 1	0	0	0 2
l. 1	0	0	0 1	0	0	0 1	0	0	0 2	0	0	0 2
2	0	0	0 3	0	0	0 3	0	0	0 3	0	0	0 3
3	0	0	1	0	0	1	0	0	1 1	0	0	1 1
4	0	0	1 2	0	0	1 2	0	0	1 3	0	0	1 3
5	0	0	1 3	0	0	2	0	0	2 1	0	0	2 1
6	0	0	2 1	0	0	2 1	0	0	2 2	0	0	2 3
7	0	0	2 2	0	0	2 3	0	0	3	0	0	3 1
8	0	0	2 3	0	0	3 1	0	0	3 2	0	0	3 3
9	0	0	3 1	0	0	3 2	0	0	4	0	0	4 1
10	0	0	3 2	0	0	4	0	0	4 1	0	0	4 3
20	0	0	7	0	0	7 3	0	0	8 3	0	0	9 2
30	0	0	10 2	0	0	11 3	0	1	1 1	0	1	2 1
40	0	1	2 1	0	1	3 3	0	1	5 1	0	1	6 3
50	0	1	5 3	0	1	7 3	0	1	9 2	0	1	11 2
60	0	1	9 1	0	1	11 2	0	2	2	0	2	4 2
70	0	2	0 3	0	2	3 3	0	2	6 1	0	2	9 1
80	0	2	4 1	0	2	7 2	0	2	10 2	0	3	1 3
90	0	2	8	0	2	11 2	0	3	3	0	3	6 2
100	0	2	11 2	0	3	3 2	0	3	7 1	0	3	11 1
200	0	5	11	0	6	6 3	0	7	2 3	0	7	10 2
300	0	8	10 2	0	9	10 1	0	10	10	0	11	9 3
400	0	11	10	0	13	0 1 3	0	14	5 2	0	15	9 1
500	0	14	9 2	0	16	5 1	0	18	1	0	19	8 3
600	0	17	9	0	19	8 3	1	1	8 1	1	3	8
700	1	0	8 2	1	3	0	1	5	3 3	1	7	7 1
800	1	3	8	1	6	3 3	1	8	11	1	11	6 3
900	1	6	7 2	1	9	7	1	12	6 2	1	15	6
1000	1	9	7	1	12	10 2	1	16	2	1	19	5 1

A TABLE of Simple Interest at 6 per Cent.

Principal	13 Days.				14 Days.				15 Days.			
	l.	s.	d.	q.	l.	s.	d.	q.	l.	s.	d.	q.
19	0	0	0		0	0	0	1	0	0	0	1
10	0	0	0	1	0	0	0	1	0	0	0	1
11	0	0	0	1	0	0	0	1	0	0	0	1
12	0	0	0	1	0	0	0	1	0	0	0	1
13	0	0	0	1	0	0	0	1	0	0	0	1
14	0	0	0	1	0	0	0	1	0	0	0	1
15	0	0	0	1	0	0	0	1	0	0	0	1
16	0	0	0	1	0	0	0	1	0	0	0	2
17	0	0	0	1	0	0	0	2	0	0	0	2
18	0	0	0	2	0	0	0	2	0	0	0	2
19	0	0	0	2	0	0	0	2	0	0	0	2
20	0	0	0	2	0	0	0	2	0	0	0	2
2	0	0	0	1	0	0	0	1	0	0	0	1
3	0	0	0	1	0	0	0	1	0	0	0	1
4	0	0	0	2	0	0	0	2	0	0	0	2
5	0	0	0	2	0	0	0	2	0	0	0	3
6	0	0	0	3	0	0	0	3	0	0	0	3
7	0	0	0	3	0	0	0	4	0	0	0	4
8	0	0	0	4	0	0	0	4	0	0	0	4
9	0	0	0	4	0	0	0	5	0	0	0	5
10	0	0	0	5	0	0	0	5	0	0	0	5
20	0	0	0	10	0	0	0	11	0	0	0	11
30	0	1	3	2	0	1	4	3	0	1	5	3
40	0	1	8	2	0	1	10	0	0	1	1	2
50	0	2	1	2	0	2	3	3	0	2	5	2
60	0	2	6	3	0	2	9	1	0	2	1	2
70	0	2	1	3	0	3	2	3	0	3	5	1
80	0	3	5	0	3	8	1	0	3	1	1	1
90	0	3	10	1	0	4	1	3	0	4	5	1
100	0	4	3	1	0	4	7	1	0	4	1	1
200	0	8	6	2	0	9	2	2	0	9	10	1
300	0	12	9	3	0	13	9	2	0	14	9	2
400	0	17	1	0	0	18	5	0	0	19	8	3
500	1	1	4	2	1	3	0	0	1	4	7	3
600	1	5	7	3	1	7	7	1	1	9	7	0
700	1	9	10	3	1	12	2	2	1	14	6	1
800	1	14	2	1	1	16	9	3	1	19	5	2
900	1	18	5	2	2	1	5	0	2	4	4	2
1000	2	2	8	3	2	6	0	1	2	9	3	3

A TABLE of Simple Interest at 6 per Cent.

Prin- cipal.	30 Days.	1 Month.	2 Months.	3 Months.	4 Months.
l. s. d. q.	l. s. d. q.	l. s. d. q.	l. s. d. q.	l. s. d. q.	l. s. d. q.
5	0 0 0 1	0 0 0 2	0 0 0 3	0 0 1 0	0 0 1 0
6	0 0 0 1	0 0 0 2	0 0 1 0	0 0 1 1	0 0 1 1
7	0 0 0 1	0 0 0 3	0 0 1 1	0 0 1 2	0 0 1 2
8	0 0 0 2	0 0 0 3	0 0 1 1	0 0 1 3	0 0 1 3
9	0 0 0 2	0 0 1	0 0 1 2	0 0 2 0	0 0 2 0
10	0 0 0 2	0 0 1 1	0 0 1 3	0 0 2 1	0 0 2 1
11	0 0 0 2	0 0 1 1	0 0 2 0	0 0 2 2	0 0 2 2
12	0 0 0 3	0 0 1 2	0 0 2 0	0 0 2 3	0 0 2 3
13	0 0 0 3	0 0 1 2	0 0 2 1	0 0 3 0	0 0 3 0
14	0 0 0 3	0 0 1 2	0 0 2 2	0 0 3 1	0 0 3 1
15	0 0 0 3	0 0 1 3	0 0 2 3	0 0 3 2	0 0 3 2
16	0 0 0 3	0 0 1 3	0 0 2 3	0 0 3 3	0 0 3 3
17	0 0 1	0 0 2	0 0 3 0	0 0 4 0	0 0 4 0
18	0 0 1	0 0 2	0 0 3 1	0 0 4 1	0 0 4 1
19	0 0 1	0 0 2 1	0 0 3 2	0 0 4 2	0 0 4 2
20	0 0 1 1	0 0 2 1	0 0 3 2	0 0 4 3	0 0 4 3
21	0 0 2 1	0 0 4 3	0 0 7 0	0 0 9 2	0 0 9 2
30	0 0 3 2	0 0 7	0 0 10 2	0 1 2 1	0 1 2 1
40	0 0 4 3	0 0 9 2	0 1 2 1	0 1 6 3	0 1 6 3
50	0 0 5 3	0 0 11 3	0 1 5 3	0 1 11 2	0 1 11 2
60	0 0 7	0 1 2 1	0 1 9 1	0 2 4 2	0 2 4 2
70	0 0 8 1	0 1 4 2	0 2 0 3	0 2 9 1	0 2 9 1
80	0 0 9 2	0 1 6 3	0 2 4 2	0 3 1 3	0 3 1 3
90	0 0 10 2	0 1 9 1	0 2 8 0	0 3 6 2	0 3 6 2
100	0 0 11 3	0 1 11 3	0 2 11 2	0 3 11 1	0 3 11 1
200	0 1 11 2	0 3 11 1	0 5 11 0	0 7 10 3	0 7 10 3
300	0 2 11 2	0 5 11	0 8 10 2	0 11 10 0	0 11 10 0
400	0 3 11 1	0 7 10 3	0 11 10 0	0 15 9 1	0 15 9 1
500	0 4 11	0 9 10 2	0 14 9 2	0 19 8 3	0 19 8 3
600	0 5 11	0 11 10 0	0 17 9 0	1 3 8 0	1 3 8 0
700	0 6 10 3	0 13 9 3	1 0 8 2	1 7 7 1	1 7 7 1
800	0 7 10 2	0 15 9 1	1 3 8 0	1 11 6 3	1 11 6 3
900	0 8 10 2	0 17 9 0	1 6 7 2	1 15 6 0	1 15 6 0
1000	0 9 10 1	0 19 8 3	1 9 7 0	1 19 5 2	1 19 5 2
2000	0 19 8 3	1 19 5 1	2 19 2 0	3 18 10 3	3 18 10 3
3000	1 9 7	2 19 2 0	4 8 9 1	5 18 4 1	5 18 4 1
4000	1 19 5 2	3 18 10 3	5 18 4 1	7 17 9 3	7 17 9 3
5000	2 9 3 3	4 18 7 2	7 7 11 1	9 17 3 1	9 17 3 1
6000	2 19 2	5 18 4 1	8 17 6 1	11 16 8 2	11 16 8 2
7000	3 9 0 2	6 18 1 0	10 7 1 2	13 16 2 0	13 16 2 0
8000	3 18 10 3	7 17 9 2	11 16 8 2	15 15 7 1	15 15 7 1
9000	4 8 9 1	8 17 6 1	13 6 3 3	17 15 0 3	17 15 0 3
10000	4 18 7 2	9 17 3 1	14 15 10 3	19 14 6 1	19 14 6 1

A TABLE of Simple Interest at 6 per Cent.

Principal.	5 Months. l. s. d. q.	6 Months. l. s. d. q.	7 Months. l. s. d. q.	8 Months. l. s. d. q.
5	0 0 12	0 0 13	0 0 20	0 0 21
6	0 0 13	0 0 20	0 0 21	0 0 23
7	0 0 20	0 0 22	0 0 30	0 0 31
8	0 0 21	0 0 23	0 0 31	0 0 33
9	0 0 23	0 0 31	0 0 33	0 0 41
10	0 0 30	0 0 32	0 0 41	0 0 43
11	0 0 31	0 0 40	0 0 42	0 0 51
12	0 0 32	0 0 41	0 0 50	0 0 52
13	0 0 33	0 0 43	0 0 51	0 0 60
14	0 0 40	0 0 50	0 0 53	0 0 62
15	0 0 41	0 0 51	0 0 60	0 0 70
16	0 0 43	0 0 52	0 0 62	0 0 72
17	0 0 50	0 0 60	0 0 70	0 0 80
18	0 0 51	0 0 61	0 0 72	0 0 82
19	0 0 52	0 0 63	0 0 73	0 0 90
l. 1	0 0 53	0 0 70	0 0 81	0 0 92
2	0 0 11 3	0 1 21	0 1 42	0 1 63
3	0 1 53	0 1 91	0 2 03	0 2 42
4	0 1 11 3	0 2 42	0 2 91	0 3 13
5	0 2 53	0 2 11 2	0 3 52	0 3 11 1
6	0 2 11 2	0 3 62	0 4 13	0 4 83
7	0 3 52	0 4 13	0 4 100	0 5 65
8	0 3 11 1	0 4 83	0 5 61	0 6 33
9	0 4 51	0 5 40	0 6 22	0 7 11
10	0 4 11 0	0 5 11 0	0 6 103	0 7 103
20	0 9 10 1	0 11 10 0	0 13 92	0 15 91
30	0 14 92	0 17 90	1 0 82	1 3 80
40	0 19 83	1 3 80	1 7 71	1 11 63
50	1 4 73	1 9 70	1 14 61	1 19 51
60	1 9 70	1 15 60	2 1 50	2 7 40
70	1 14 61	2 1 50	2 8 40	2 15 23
80	1 19 51	2 7 40	2 15 23	3 3 12
90	2 4 42	2 13 30	3 2 12	3 11 00
100	2 9 33	2 19 20	3 9 02	3 18 103
200	4 8 72	5 18 40	6 18 10	7 17 93
300	7 7 11 1	8 17 61	10 7 12	11 16 82
400	9 17 31	11 16 82	13 16 20	15 15 71
500	12 6 63	14 15 103	17 5 22	19 14 61
600	14 15 102	17 15 03	20 14 30	23 13 50
700	17 5 22	20 14 30	24 3 32	27 12 40
800	19 14 61	23 13 50	27 12 40	31 11 23
900	22 3 100	26 12 71	31 1 42	35 10 12
1000	24 13 13	29 11 91	34 10 50	39 9 02

A TABLE of Simple Interest at 6 per Cent.

s.	9 Months.			10 Months,			11 Months,			1 Year.		
	l.	s.	d.	q.	l.	s.	d.	q.	l.	s.	d.	q.
5	0	0	2	3	0	0	3	0	0	0	3	3
6	0	0	3	1	0	0	3	2	0	0	4	1
7	0	0	3	3	0	0	4	1	0	0	4	2
8	0	0	4	1	0	0	4	3	0	0	5	1
9	0	0	4	3	0	0	5	1	0	0	5	3
10	0	0	5	1	0	0	5	3	0	0	6	2
11	0	0	5	3	0	0	6	2	0	0	7	0
12	0	0	6	1	0	0	7	0	0	0	7	3
13	0	0	6	3	0	0	7	2	0	0	8	2
14	0	0	7	2	0	0	8	1	0	0	9	0
15	0	0	8	0	0	0	8	3	0	0	9	3
16	0	0	8	2	0	0	9	1	0	0	10	2
17	0	0	9	0	0	0	10	0	0	0	11	0
18	0	0	9	2	0	0	10	2	0	0	11	3
19	0	0	10	0	0	0	11	0	0	1	0	1
20	0	0	10	2	0	0	11	3	0	1	1	0
21	0	0	11	0	0	1	1	0	0	1	2	2
22	0	1	9	1	0	1	11	3	0	2	2	0
23	0	2	8	0	0	2	11	2	0	3	3	0
24	0	3	6	2	0	3	11	1	0	4	4	0
25	0	4	5	1	0	4	11	2	0	5	5	0
26	0	5	4	0	0	5	11	0	0	6	6	0
27	0	6	2	2	0	6	10	3	0	7	7	0
28	0	7	1	1	0	7	10	3	0	8	8	0
29	0	7	11	3	0	8	10	2	0	9	9	0
30	0	8	10	2	0	9	10	1	0	10	10	0
31	0	17	9	0	0	19	8	3	1	1	8	1
32	1	6	7	2	1	9	7	0	1	12	6	2
33	1	15	6	0	1	19	5	2	2	3	4	2
34	2	4	4	2	2	9	3	3	2	14	3	0
35	2	13	3	0	2	19	2	0	3	5	1	0
36	3	2	1	2	3	9	0	2	3	15	11	1
37	3	11	0	0	3	18	10	3	4	6	9	2
38	3	19	10	3	4	8	9	1	4	17	7	3
39	4	8	9	1	4	18	7	2	5	8	5	3
40	8	17	6	1	9	17	3	0	10	16	11	2
41	13	6	3	2	14	15	10	2	16	5	5	2
42	17	15	0	0	19	14	6	1	21	13	11	1
43	22	3	10	3	24	13	1	3	27	2	5	1
44	26	12	7	1	29	11	9	1	32	10	11	1
45	31	1	4	2	34	10	5	0	37	19	5	1
46	35	10	1	2	39	9	0	2	43	7	11	0
47	39	18	10	3	44	7	8	0	48	16	5	0
48	44	7	8	0	49	6	3	2	54	4	10	3
49												

A TABLE of Compound Interest at 6 per Cent.

Years.	The A- mount of 1 Pound.	The pre- sent worth of 1 pound.	The A- mount of 1 pound An- nuity.	The pre- sent worth of 1 pound Annuity.	The An- nuity that 1 l. will Purchase.
1	1.06	.943396	1.00000	0.943396	1.06000
2	1.1236	.889996	2.06000	1.833392	.94543
3	1.19101	.839619	3.18360	2.673012	.87411
4	1.26247	.792093	4.37461	3.465105	.81559
5	1.33833	.747258	5.63709	4.212363	.76739
6	1.41851	.704960	6.97531	4.917324	.72836
7	1.50363	.665057	8.39383	5.582381	.69513
8	1.59384	.627412	9.89746	6.209792	.66603
9	1.68947	.591898	11.49131	6.801691	.64072
10	1.79084	.558391	13.18079	7.360086	.61786
11	1.89829	.526787	14.97164	7.886873	.59679
12	2.01219	.496969	16.86994	8.383843	.57727
13	2.13292	.468839	18.88213	8.852682	.55926
14	2.26091	.442300	21.01506	9.294983	.54258
15	2.39655	.417265	23.27596	9.712248	.52706
16	2.54035	.393646	25.67252	10.105903	.51269
17	2.69277	.371364	28.21287	10.477266	.49944
18	2.85433	.350343	30.90565	10.827602	.48723
19	3.02559	.330512	33.75999	11.158115	.47596
20	3.20713	.311804	36.78559	11.469920	.46551
21	3.39956	.294154	39.99272	11.754200	.45580
22	3.60353	.277505	43.39228	12.041466	.44684
23	3.81975	.261797	46.99582	12.303377	.43862
24	4.04893	.246978	50.81557	12.550356	.43107
25	4.29187	.232998	54.86451	12.783354	.42422
26	4.54938	.219810	59.15638	13.003164	.41799
27	4.82234	.207367	63.70576	13.210532	.41226
28	5.11178	.195630	68.52810	13.406162	.40704
29	5.41838	.184556	73.63979	13.590719	.40231
30	5.74349	.174110	79.05818	13.764829	.39806
31	6.08807	.164255	84.80167	13.929166	.39427
32	6.46035	.154956	90.88977	14.084050	.39092
33	6.84068	.146186	97.34316	14.230227	.38799
34	7.25098	.137946	104.18375	14.367666	.38546
35	7.68605	.130105	111.43477	14.531583	.38321
40	10.28563	.097222	154.76196	15.046300	.36646
45	13.76458	.072660	212.74299	15.455666	.35470
50	18.42003	.054287	290.33382	15.761883	.34644
55	24.65005	.040566	394.16748	15.990566	.34023
60	32.98719	.030313	533.11981	16.161450	.33587

A TABLE of Compound Interest at 10 per Cent.

Years.	The A- mount of 1 Pound.	The pre- sent worth of 1 pound.	The A- mount of 1 pound An- nuity.	The pre- sent worth of 1 pound Annuity.	The An- nuity that 1 l. will Purchase.
1	1.1	.90909	1.	0.90909	1.1
2	1.21	.826446	2.1	1.73553	.57619
3	1.331	.751314	3.31	2.48685	.40211
4	1.4641	.683013	4.641	3.16986	.31947
5	1.61051	.620921	6.1051	3.79078	.26379
6	1.77156	.564574	7.71561	4.35526	.2295
7	1.94871	.513158	9.48717	4.86841	.20545
8	2.14358	.466507	11.43588	5.33492	.18744
9	2.35794	.424097	13.57947	5.75901	.17364
10	2.59374	.385543	15.93742	6.14456	.16274
11	2.85311	.350494	18.53116	6.49506	.15396
12	3.13842	.31863	21.38428	6.81366	.14679
13	3.45217	.289664	24.52271	7.10335	.14077
14	3.79749	.263331	27.97498	7.36668	.13574
15	4.17724	.239392	31.77248	7.60608	.13147
16	4.59497	.217629	35.94972	7.82371	.12781
17	5.05447	.197844	40.5447	8.02155	.12466
18	5.55991	.179858	45.59917	8.20141	.12192
19	6.1159	.163508	51.15909	8.36492	.11954
20	6.72749	.148643	57.27499	8.51356	.11745
21	7.40024	.13513	64.00249	8.64869	.11562
22	8.14027	.122846	71.40274	8.77154	.114
23	8.9543	.111678	79.54302	8.88329	.11257
24	9.84973	.101525	88.49732	8.98474	.11129
25	10.8347	.092296	98.34705	9.07704	.11016
26	11.91818	.083905	109.18176	9.16094	.10915
27	13.10999	.076277	121.09994	9.23722	.10825
28	14.42099	.069343	134.20993	9.30656	.10745
29	15.86309	.063039	148.63092	9.3696	.10672
30	17.4494	.057308	164.49402	9.42691	.10607
31	18.49636	.052098	181.94342	9.479	.10558
32	19.60614	.047361	200.43978	9.52636	.10497
33	20.78251	.043055	220.04593	9.56941	.10449
34	22.02946	.03914	240.82844	9.60856	.10407
35	23.35123	.035582	262.85790	9.64413	.10369
36	24.7523	.032347	286.20914	9.67657	.10334
37	26.23744	.029406	310.96144	9.70598	.10302
38	27.81168	.026733	337.19888	9.73271	.10274
39	29.48038	.024302	365.01057	9.75701	.10249
40	31.24921	.022092	394.49095	9.77911	.10225

The use of the Table of Simple Interest.

This Table serves whether the Interest is to be Calculated for a certain number of Days, or even Twelfths or fourth parts of a Year, which those Tables cannot, which are said to be calculated for 1, 2, 3, &c. Months, and at the same time never shew what they mean by those Months; it is well known our Months are different in length, and as for their even Twelfth parts of a Year, there is no such time taken notice of in any Kalender or Computation of time: therefore I think where the Interest Tables are for Months: there ought some mention to be made what Months they mean, as *January, February, &c.* because they differ much, or else to call so many days a Month, which I have done for the more easie computing the exact Interest for any Number of Days; but I have said more of this in my Supplement to *Comes Commerci*: I shall proceed here to shew how to find the Interest for any Number of Years, Days, or Quarters of Years.

Proposition. 1.] *To find the Interest of any Summ for any Number of Months, or Days, as from any Day of one Month to any of another Month: as the Interest of l. 500 from January 21, to October 11th. following.*

The intire Months in this time are 8, of which there are 4 that have 31 Days therein, and 1 that has but 28; which is 2 short of 30, which 2 deducted from the 4, there rests 2 Days above 8 Months, and those 2 added to the 11 Days in *January*, and the 10 in *October* the Summ is 8 Months and 23 Days. So the Interest by the Table of

	l.	s.	d.
<i>l. 500 for 8 Months of 30 Days each, is</i>	19	14	06 $\frac{1}{2}$
<i>and for 15 and 8, or 23 Days</i>	=	=	1 17 09 $\frac{1}{2}$

The Summ or Answer = 21 12 03 $\frac{3}{4}$

And that you may the better know the Days in each Month, and compute the odd Days above 30, take the same as follows.

1

January

The use of the Tables of Compound Interest.

Case 1. To find the Amount of any Summ forborn any Number of Years, as if a Testator bequeath to a Legatee *l* 500 to be paid him at the end of 21 years, with the utmost improvement of Interest at 6 per Cent. The Question is what must then be paid.

Case 2. To find the present worth of any Summ due at the end of any Number of years to come. As suppose a Legatee is minded to sell for present Money, the Summ of *l* 1699 : 15 : 7½ which was left him payable at the end of 21 years, what is it worth presently.

Case 3. To find the Amount of any Annuity forborn any Number of Years : as if an Infant has an Annuity or Rent charge of 30 *l* per Ann. payable out of the Rent of certain Lands, to be paid him at the end of 21 years, with the utmost improvement for that time by the person then in possession, what will that Annuity be then worth?

Case 4. To find the present worth of the Reversion,

- | | | |
|----------------------------|---|--------------------|
| 1. Of an Estate in Land, | } | in Fee |
| 2. Of an Estate in Houses, | | |
| 3. Of an Estate in Land, | } | for a time limited |
| 4. Of an Estate in Houses, | | |

Case 5. To find the Annuity that any Summ will purchase, either of Land or Houses in present, or in Reversion, wherein is shewed to Fine off Rent of Land or Houses, &c.

The Answer to Case 1.

Look in the Table of Compound Interest at 6 per Cent. and against 21 in the Collum of Years you shall find *l* 3.39956 which is the Amount of 1 *l*. for 21 Years, and being multiplied by 500 the Product is *l* 1699.78 or *l* 1699 : 15 : 7½.

See the Work.

$$\begin{array}{r}
 3.39956 \\
 500 \\
 \hline
 1699.78000
 \end{array}$$

Answer to Case 2.

The solving this Case proves the truth of the last, and it is done by the second Table from that of Years, for against 21 Years in the Table of 6 per Cent. you have in the Column of [the present worth of 1 *l*.] .294154 which multiply by the Principal 1699.78 and the Product will be *l* 500, the present worth of 1699.78 due 21 Years hence.

I 2

Answer

*Proof of
the last
Case.*

Answer to Case 3.

In the third Column from that of years in the Table of 6 per Cent. you have against 21 years, $l. 39.99272$ the Amount of 1 *l.* Annuity forborn 21 Years, which multiplied the $l. 30$ the Annuity given, the Product will be $l. 1199. 15: 7\frac{1}{2}$. See the operation where the value of the Decimals $.781 l.$ is (the 60 being not considerable) $15 s. 7\frac{1}{2} d.$

$$\begin{array}{r} 39.99272 \\ \times 30 \\ \hline 1199.78160 \end{array}$$

Answers and Examples to Case 4.

The Common Value of Land, or Houses for Ever, or for Lives, or Years.

The value of an Estate in Land, Fee-Simple is commonly about 20 Years Purchase. And of an Estate in Houses if well Built, is but worth in Fee about 12 Years Purchase; because of the great Charge that attends the maintaining and upholding thereof. Also in the Purchasing of Leases 7 Years, is commonly reckoned Equivalent to 1 Life, 14 to 2, and 21 Years to 3 Lives; but 3 Lives in a Lease are generally much more advantageous to the Purchaser than 21 Years, though he pay a Years Purchase for Renewing a Life when it falls.

The Purchase of an Estate in Land.

Years. Months.

For 1 Life, or 7 Years is 5 : 7 Purchase.

For 2 Lives, or 14 Year = 9 : 3

For 3 Lives, or 21 Years = 11 : 9

The Purchase of an Estate in Houses.

Years. Months.

For 1 Life, or 7 Years is = 4 : 10 Purchase.

2 Lives or 14 Years = 7 : 4

3. Lives, or 21 Years = 8 : 7

For

For they that Purchase Land expect but 6 *per Cent.* for their Money, but those that lay out their Money in the Purchase of Houses expect 10 *per Cent.* for their Money, for the reason aforesaid.

This being premised, I shall shew the method of Purchasing the Reversion of any Estate, as

1. Admit I have 15 Years to come in a Lease of Land, for which I pay 80 *l. per Ann.* which I am willing to Purchase in Fee, after the Expiration of my Lease; What is such an Estate worth in present Money?

To answer this, I suppose the Land in Fee to be worth 20 Years Purchase, *i. e.* 20 times 80 Pounds or *l.* 1600.

Then I consider what is the present worth of 180 *per Ann.* to continue 15 Years, which (by the help of the 4th Column from that of Years in the Compound Interest Table of 6 *per Cent.* I find *l.* 823 : 00 : 4 $\frac{3}{4}$.

For against 15 Years in that Column is the present worth of 1 *l.* Annuity to continue 15 Years.

Which Multiply by ——— 80

and the Product is — *l.* 776.979840
1600.0

which is the present worth of 180 *per Ann.* to continue 15 Years, which deducted from the *l.* 1600 the rem. is *l.* 823 020 16 which must be given presently for the Reversion of the Land in Fee after the Expiration of the Lease.

2. Or if this 180 *per Ann.* be in Houses, then the Reversion in Fee of Houses Simply after 15 Years is expired, is worth in present Money *l.* 351 : 10 : 3

For the value of 80 *l. per Ann.* in Fee, is here 12 Years Purchase, *i. e.* 12 times 80, or 960 *l.*

And the present worth of 1 *l. per Ann.* at 10 *per Cent.* in the Compound Interest Table, against 15 Years is — *l.* 7.60608

Which multiplied by the Annual Rent. 80

The product is ——— *l.* 608.48640
960

which product is the present worth of 180 *per Ann.* in Houses to continue 15 Years, which deducted from the Purchase in Fee, *l.* 960, the remainder is — *l.* 351.5136 which must be given presently for the Reversion of that Estate in Houses.

3; But.

3. But if this Tenant is minded to augment the time in his Lease, as suppose he would Purchase with present Money 20 Years next after the 15 that is in his Lease, what shall he give.

A Rule for purchasing the Reversion of land or houses for a time limited.

Add the 15 and the 20 together, and they make 35 Years, then find by the 4th Column in the Compound Interest Table (of 6 per Cent. if the Estate be in Land, or of 10 per Cent. if it be in Houses) the present worth of 1 l. per Ann. to continue 35 Years, and Multiplying that by 80, you have the present worth of 80 l. per Ann. to continue 35 Years respectively.

Then find by the very same method the present worth of 1. 80 to continue but 15 Years, and deduct that from the present worth for 35 Years, and the remainder is the Summ to be paid presently for 20 Years Reversion, after the 15 Years Lease is expired. See the Work.

15 more 20 is = 35 Years (suppose this to be Land).

Example for Land.

The present worth of 1 l. per Ann. to continue 35 Years is $l. 14.531583$
 Multiply by — Years 80
 and the present worth of 80 l. per Ann. to }
 continue 35 Years is ————— } 1162.526640
 now the present worth of 1. 80 per Ann. to cont. 15 years is 776.97984

which deducted the Remainder is the Summ to be paid presently for 20 years Annuity of 80 l. after the expiration of 15 years; which is $l. 385.5468$

Or if the Estate be in Houses.

Example for Houses.

Then by the Compound Interest Table of 10 per Cent.

The present worth of 1 l. per Ann. to contin. 35 years is $l. 9.64413$
 Multiply by years 80

and you'll find the present worth of 1. 80 per Ann. }
 to continue 35 years ————— } $l. 771.53040$
 the present worth of 80 l. per Ann. to cont. 15 years is $l. 608.4864$
 which deducted, the remainder is $l. 163 : 00 : 10\frac{1}{2}$
 which is the Summ to be paid presently for an Annuity }
 of 1. 80 Rent of Houses, for 20 Years next after the 15 } 163.044
 Years is expired. ————— }

4. A hath a Lease of a House of 50 l. per Ann. to continue 15 years, and B hath one of 40 l. per Ann. to continue 35 years, and they agree to make an exchange; The Question is what one must give the other in present Money?
 To

To solve this, find the present worth of *A* his Lease, of 50 *l.* per Ann. to continue 15 years, which is 1. 380: 6: 1

Also the present worth of *B* his 40 *l.* per Ann. to continue 35 years is 1. 385: 15: 3½

Here it appears that *A* must give *B* 1. 5: 9: 2½ in hand, the present worth of his Estate being so much more than that of *A*'s. Difference 1. 5: 9: 2½

Answers and Examples to Case 5.

To find what Annuity to continue any Number of years, any Summ will Purchase, is performed by help of the 5th Column, from that of Years.

Example 1.] There is a Lease of a House to be Lett for 31 years, of 1. 60 per Ann. and 1. 200 Fine; but a Tennant for the advantage of his Wife in case of his own death, would pay the less Rent, and 1500 Fine: The Question is what Rent he must pay?

Here the Money advanced by the Tennant in the Fine is 1. 300, therefore the Annuity that 1. 300 will Purchase to continue 31 years, must be deducted out of the Annuity 60 *l.* and the Remainder must be paid and the Fine 1. 500. *A Rule to Fine off Rent of Houses.*

By the Table of 10 per Cent. 1. 1 will Purchase } 1. 10558
per Ann. for 31 years. _____

Which Multiplied by = 300

The Product is = 1. 31.67400

Which 1. 31.674 deducted from the Annuity 60 *l.* the remainder is 28. 326, or 1. 28: 06: 6½, which is the Rent to be paid for 31 years, the Purchaser paying 1. 500 Fine.

Examp. 2.] There is a Lease of a House to be Lett for 31 years, for 1. 28: 6: 6½ per Ann and 1. 500 Fine; but the Lessee being a Trader newly set up, and having occasion for present Money, is willing to pay the greater Rent, so that he may pay but 1. 200 Fine; How must his Rent be encreased, to make the Bargain Equivalent to the said 1. 28: 6: 6½ Rent, and 1. 500 Fine? *How a Fine may be less'n'd by a greater Rent.*

Here the difference, or abatement of the Fine being 1. 300, therefore the Annuity that 1. 300 will Purchase, to continue 31 years at 10 per Cent. i. e. 1. 31: 13: 5¼ being added to the proposed Annual Rent, 1. 28: 6: 6½, the Summ is 1. 60 the Rent, to be paid for 31 years and 1. 200 Fine; which proves the last Question. *Examp.*

Examp. 3.] There is a Lease of a House for 31 years of $l. 28 : 6 : 6\frac{1}{2}$ per Ann. Rent, and $l. 500$ Fine; but the Lessee is willing to pay $60 l.$ per Ann. Rent, that he may pay a less Fine: The Question is what Fine he ought to pay?

In this Case the difference of the Rent is $l. 31 : 13 : 5\frac{3}{4}$: so that it is plain the Lessee must pay so much the less Fine, as that Summ is, which will Purchase $l. 31 : 13 : 5\frac{3}{4}$ per Ann.

Now to know what Summ will Purchase, or the present worth of that Annuity, to continue 31 years; look in the 4th Column from that of Years, and against 31 you'll find 9.479, which is the present worth of $1 l.$ per Ann. to continue 31 years, and being multiplied by the Annual Rent, $l. 31 : 13 : 5\frac{3}{4}$, or 31674, the product is $l. 300$; which deducted from the Fine proposed, *i. e.* $l. 500$, the remainder is $l. 200$, and so much Fine must be paid, and $l. 60$ per Ann. Rent, to answer the demand.

To Purchase
Annuities
in Reversion.

Examp. 4.] A Gentleman having received his Fortune which is $l. 1500$, and also a promise of maintainance for 5 years *Gratis*; he is minded to Purchase an Annuity with the said $l. 1500$ to continue 7 years to commence after the Expiration of the 5; the Question is what Annuity to continue that 7 years, the $l. 1500$ will Purchase?

To answer this, first find the amount of $l. 1500$ for 5 years Compound Interest, at 10 per Cent. which is $l. 2415 : 15 : 3\frac{1}{2}$.

Then find what Annuity $l. 2415 : 15 : 3\frac{1}{2}$ will Purchase to continue 7 years, which is $l. 496 : 6 : 4\frac{1}{2}$ per Ann. the answer; found by multiplying the 5th Tabular N^o from that of years, in the Table of 10 per Cent. against 7 years, *i. e.* .20545 by $l. 2415 : 15 : 3\frac{1}{2}$, or $l. 2415.765$.

Note, That though I have done this at 10 per Cent. yet Annuities are most properly computed at 6 per Cent. by the 1st Table of Compound Interest.

To Calculate any of the Numbers in the Tables of Compound Interest.

1. The Column next that of Years is made thus, for 6 per Cent.
 1. As 100. to 106 :: so 1. to 1.06 the 1st N^o.
 2. As 100. 106 :: 1.06. 1.1236 the 2d. N^o. and so the rest.
2. The second Column from that of Years is made thus.
 1. As 106. 100 :: 1. .943396 — the 1st. N^o.
 2. As 106. 100 :: .943396. .889996 — the 2d. N^o. &c.

3. The

3. *The Third Column from that of Years is made thus:*
 1. The first Number is always 1.
 2. The second in this Column is the Summ of the first in the first and third Columns, &c.
4. *The Fourth Column from that of Years is thus made:*
 1. The first is always the same with the first in the second Column.
 2. The second in this, is the Summ of the first in this and the second in the second Column, &c.
5. *The Fifth Column from that of Years is thus Calculated:*
 Divide a Unit by any of the Numbers in the fourth Column, and the Quotient is the Respective N°. in this fifth Column.
 1. The first Number in this, is the first in the first Column.
 2. As 1833392. 1 : : 1. 54543 the second Number in this fifth Column, &c.

CHAP. XI.

Of the Extraction of the Square and Cube Roots.

SECT. 1. *The Extraction of the Square-Root.*

1. **T**HE Extraction of the Square-Root is the finding a Geometrical mean proportional between 1 and the Number given, *i. e.* it is the finding out such a Number, as being multiplied by it self produces the Number given.

2. Hence it follows, that every Number cannot have its Root Extracted, because every Number cannot be produced by Multiplying another Number in it self; but all Numbers are either compleat or imperfect.

3. A compleat Square-Number is one that can have its Root extracted without a Remainder, being the Product of any Number multiplied in it self, as 4, 9, 16, 25, &c.

4. An imperfect Square-Number is one that cannot have its Root extracted without a remainder, as not being the product of any Number whatsoever multiplied in it self, as 2, 3, 5, 6, 7, 8, 10, 11, 12, &c.

K

5. The

*Compleat
and imper-
fect Square
Numbers
what.*

68 The Extraction of the Square-Root.

What the
Root and
the Square
of any
Number is.

5. The Number so to be multiplied by it self is called the Root, and the Product arising from the Multiplication is called the Square, as the Line ab , cd , ad , or bc is the Root, or side of the Square, which is here 3, and the 9 little Squares is the Product of 3 (any of the sides) multiplied in it self, which is the Square of the side or Root; the use of all which will appear in the next Chapter.



6. A TABLE of the Squares of all the 9 Digits or Roots take as follows, which will be useful in the Work of Extraction following.

Roots =	1	2	3	4	5	6	7	8	9
Squares =	1	4	9	16	25	36	49	64	81

An Ex-
ample of
Extracting
the Root of
a Complex
Square
Number.

7. If you would find the Square-Root of any whole Number as of 1550025, you must first make a point over Units place, and so over every second toward the Left Hand as you see in the Example following; where observe, that you will always have as many places in the Root as there are points, and the 4 points divides as it were the Number given into 4 parts or branches.

2. Then consider what Number being multiplied by it self will produce a Square next to, and less than the first branch toward the Left Hand, which by the foregoing Table of Roots and their Squares is 1, which therefore put in the Root as in a Quotient.

. . . . The Root.
The Square. 1550025 (1245.

$$\begin{array}{rcl}
 \text{Divisors } \left\{ \begin{array}{l} 22 \) \ 55 \quad = 1. \text{ Divid.} \\ 244 \) \ 1100 \quad = 2. \text{ Divid.} \\ 2485 \) \ 12425 \quad = 3. \text{ Divid.} \end{array} \right. \\
 \hline
 \text{0 remainder.} \\
 \text{3. Square}
 \end{array}$$

The Extraction of the Square-Root. 69

3. Square (or Multiply by itself) the figure so put in the Quotient.
 4. Deduct that Square from the said first branch (1) toward the left hand, and the remainder is (0).

5. Bring down the next branch 55 (as you see in the Example) which must have been placed toward the Right Hand of the remainder had there been any.

6. Double the Figure (1) last put in the Root, and put that 2 like a Divisor whereby to divide the last branch 55.

7. See always how often you can have the figure so double (as here 2) in all the Figures except that in Units place of the dividend (here 5) and put the answer in the Root (in this Example 2) which put also toward the Right Hand the Divisor 2.

8. Multiply the Divisor by the said Figure last put in that Root as here 2 times 2 is 4, from 5 rest 1, and 2 times 2 is 4, from 5 rests 1, so there remains (11).

9. To that remainder (11 bring down the next branch (00) and you have (1100) for a new dividend.

10. Then double the Figure in Units place of your last Divisor, and if that double exceeds a Digit, add the 10 to the Figure in tens place of your last Divisor, as twice 2 in the last Divisor is 4, so is the new Divisor 24.

11. By this 24 divide the second Dividend (except Units place) which is 110, as here 24 is had in 110, 4 times, which 4 put in the Root, and also in Units place of the Divisor 24 making it 244, and having deducted 4 times 244 from 1100, the remainder is 124.

12. To this remainder 124 bring down the last branch 25, and it will make 12425 for the 3 dividend.

13. Then double the Figure in Units place of the last Divisor (4) and it makes 8.

14. Then divide 1242 in the Dividend (being all but Units place) by 248, and the Quotient is 5, which put also in Units place of the last Divisor, then Multiply 2485 by 5 (last put in the Root) deducting the Product from the Dividend 12425, and the remainder is (0) so is the Square-Root 1245, which Work may be proved by Multiplying the Root 1245 in it self, for if the Work is right it will produce the Number given to have its Root extracted, viz. 1550025.

Note, That if any dividend (excluding Units place) cannot be divided by the Divisor; you must put a Cypher in the Root, as also in Units place of the Divisor, and so proceed to bring down the next branch or part of the Square N^o given.

K 2

Example

*How to
prove the
Extraction
of the
Square-
Root.*

70 The Extraction of the Square-Root.

Example 2.

What is the Square-Root of 144057642 ?

Root.

$$\begin{array}{r} 144057642 \quad (12002 \\ \hline 22) 44 = 1st. Dividend. \\ \hline 24002) 057642 \quad 2, 3 \text{ and } 4 \text{ Dividends.} \\ \hline 9638 = \text{Remains.} \end{array}$$

Here you see this Question is wrought just as the last, but only that whereas the Divisor (24) cannot be had in the part brought down, viz. (05) therefore I put (0) in the Root and Divisor, and bring down the next Branch (76) and so divide 57 by 240, putting the answer (0) in the Root, and to the Right Hand the 240, then to the 576 I bring the next Branch 42, and so divide 5764 by 2400, and the Quotient is 2, which I put in the Root, and to the Right Hand the Divisor, &c.

8. To Extract the Square-Root of a Decimal.

Decimals to have an even Number of places before Extraction.

This is done the very same way with whole Numbers, only observing that if you have not an even Number of places in the Decimal given, to make them so by adding a Cypher, or Cyphers according as you would have the Root to consist of places, for instance, What is the Square-Root of .781 ?

Example 3.

$$\begin{array}{r} .78100000 \quad (.8837 \\ \hline 168) 1410 \\ \hline 1763) 6600 \\ \hline 17667) 131100 \\ \hline 7431 \text{ Remains.} \end{array}$$

Note,

The Extraction of the Square-Root. 71

Note, That if your Decimals given have Cyphers toward the Left Hand next the point; you must for every 2 Cyphers there put one Cypher in the Quotient or Root, and so Extract the Root of the rest as is taught in whole Numbers.

How to deal with Decimals that have Cyphers next the point.

Example 4.

What is the Root of .00071? Answer .0266.
See the Work.

$$\begin{array}{r}
 .00071000 \quad (.0266 \\
 \hline
 46) \quad 310 \\
 \hline
 526) \quad 3400 \\
 \hline
 244 \text{ Remains.}
 \end{array}$$

9. So that if you would have a more Exact Root of any Imperfect Square Number, as that in the second Example above, you need only add so many couple of Cyphers toward the Right Hand the Number given, as you would have Decimal places in the Root, and work as follows, and you will have the Answer near enough the truth.

Example 5.

$$\begin{array}{r}
 144057642.000000 \quad (12002.401 = R. \\
 \hline
 22) \quad 44 \\
 \hline
 24002) \quad 057642 \\
 \hline
 240044) \quad 963800 \\
 \hline
 24004801) \quad 36240000 \\
 \hline
 12235199 \text{ Remains.}
 \end{array}$$

To Extract the Root of an imperfect Square Number, near the truth, by putting Cyphers toward the Right Hand the Number given.

Note,

The Extraction of the Square-Root.

Note, That the Decimal places being an even Number, a point will always fall over Units place of the whole Number as it ought to do.

And also *Note*, That if the Decimal in the fourth Example were joined with a whole Number, as 371 (for instance) the pointing and Root will be found as follows.

Example 6.

What is the Square-Root of 371.00071000?

*To know
how many
Decimals
must be in
the Root.*

Note, That as many points as are over the Decimal parts given, so many Decimal places must you have in the Root.

371.00071000 (19.2613

29) 271

382) 1000

3846) 23607

38521) 53110

385223) 1458900

303231 Remains.

*How to Ex-
tract the
Square Root
of a Vulgar
Fraction.*

10. The Square-Root of a Vulgar Fraction is extracted by reducing it first to its lowest Terms, and then the Roots of the Numerator and Denominator are those of the Answer. But if the Roots of both cannot be taken, the best way is to reduce the Vulgar Fraction to a Decimal, and then work as in the third and fourth Examples foregoing.

Sect.

The Extraction of the Cube-Root.

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SECT. 2.

The Extraction of the Cube-Root.

A Cube is a solid body, having six equal Geometrical Squares *Definition.* for its bounds; and to Extract the Cube-Root of any Number is to find the side of one of those Squares; so that the Cube-Root of any Number multiplied by it self, produces the content of one of the 6 Superficies, or Geometrical Squares; which Content or Square being again multiplied by the side of one of the Squares, produces the Content of the whole Cube.

For Instance, to find the Cube of the Root 4, the Square of 4 is 4 times 4, or 16, and 4 times 16 is 64, which is the Cube of 4; so is 4 the Cube-Root, and 64 the Cube.

II. Hence it follows that every Number cannot have its Cube-Root Extracted without a Remainder; because every Number cannot be produced by Multiplying another in it self, and that Product again by the Root: So that

III. Cube Numbers are either (as Squares) compleat or imperfect Cubes; for a compleat Cube Number, is that which can have *compleat* its Root extracted without a Remainder, as 8, 27, 64, 125, &c. *imperfect* But an imperfect Cube Number is that whose side or Root can never be exactly known, as 3, 4, 5, 6, 7, 9, 10, and infinite others. *Cube-Numbers what.*

A TABLE of the Cubes of the 9 Digits, or single Roots, which will be useful in Extracting the Cube-Root, take as follows.

Roots =	1	2	3	4	5	6	7	8	9
Squares =	1	4	9	16	25	36	49	64	81
Cubes =	1	8	27	64	125	216	343	512	729

IV. Admit it were required to find the Cube-Root of the compleat Cube Number, 192978125?

In order to perform this you must first point the Numbers into Branches, beginning always at Units place, and proceeding to make a point over every third, after the first, as you see in the Example following.

The

The Extraction of the Cube-Root.

The Cube Number.

1929781125 (1245 = The Cube Root.

33) 929 = first Resolvend, or Dividend.

Add { $\begin{array}{l} 3 = \text{the treble Root (1)} \\ 3 = \text{the treble Square of the Root (1)} \end{array}$

Summ = 33 the first Divisor, whereby to Divide the first Resolvend.

Add { $\begin{array}{l} 8 = \text{the Cube of 2 last put in the Root,} \\ 12 = \text{the Squ. of that 2 mult. by the last treb R.} \\ 6 = \text{the last treb. Square of the R. mult. by 2} \end{array}$

The Summ = 728 = the Subtrahend to be taken from the Ref.

4356) 201781 = the second Resolvend.

Add { $\begin{array}{l} 36 = \text{the treble Root (12)} \\ 432 = \text{the treble Square of that Root.} \end{array}$

Summ = 4356 = the 2d Divis. for the 2d Resolvend.

Add { $\begin{array}{l} 64 = \text{the Cube of 4} \\ 576 = \text{the Square of 4 in the treble Root.} \\ 1728 = \text{the treb. Sq. of the Root multipl. by 4} \end{array}$

Summ = 178624 = the Subtrah. to take from the last Ref.

46165) 23157125 = the third Resolvend.

Add { $\begin{array}{l} 372 = \text{the treble Root (124)} \\ 46128 = \text{the treble Square of that Root.} \end{array}$

Summ = 461652 the 3d Div. for the 3d Resolvend.

Add { $\begin{array}{l} 125 = \text{the Cube of 5 last put in the Root} \\ 9300 = \text{the Sq. of 5 mult. by the last treb. R.} \\ 230640 = \text{the treb. Sq. of the R. mult. by 5.} \end{array}$

Summ = 23157125 = the Subt. to take from the last Ref.

(0) Remains.

I

I have been so particular in this Example, that I shall need to give fewer directions here for performing the Operation; for you see I begin with considering what Number being Cubed will be equal to, or next less than the first branch (1) which I find to be (1), and therefore put one in the Root; so that Deducting the Cube of the Figure (1) from the first branch (1) the Remainder is (0), but if it had been something, I would bring down the next branch 929 to the Right hand thereof.

2. Here by the Work you may see that every Resolvend or Dividend must be Divided by the Summ of 3 times the then Root, and three times the Square of the Root added together.

3. In Dividing thereof you are to observe the same Rule as in the Extraction of the Square-Root, *i. e.* to take no Notice of the Units place of the Resolvend, in seeing how often the Divisor may be had in it.

4. Observe that first the Cube of the Figure (2) last put in the Root (arising from the said Division) secondly the Square of that 2 multiplied by the treble Root, and thirdly the treble Square of the Root multiplied by the said Figure put last in the Root (which is here 2) do all 3 together make a Subtrahend, or Number to be Deducted from the last preceding Dividend or Resolvend.

5. To the Remainder bring down the next branch (which is here 781) and so go on with the treble Root, &c. as before, for what follows is done just in the same method as that preceding.

6. Observe that if the Divisor cannot be had in the Resolvend (excluding the Units place of that Resolvend) then you are to put a Cypher in the Root, and bring down the next branch to the last Resolvend, and make a new Divisor of the whole Root with the Cypher last put therein.

L

Example

Example 2.

What is the Cube-Root of 8242409?

See the Work.

An Ex-
ample of
extracting
the Root of
an Imper-
fect Cube-
Number.

$$\begin{array}{r}
 8242409 \quad (202 \\
 \underline{0242} = \text{Resolvend the first:} \\
 \text{Add } \left\{ \begin{array}{l} 6 = \text{the treble Root } (2) \\ 12 = \text{the treble Square of the Root } (2) \end{array} \right. \\
 \underline{126} = \text{the Summ or Divisor, to Divide the Ref.} \\
 12060 \quad 242409 = \text{the last Resolvend, and the next branch.} \\
 \underline{60} = \text{the treble Root } (20) \\
 \underline{1200} = \text{the treble Square of the Root.} \\
 \underline{12060} = \text{the Summ or Divisor, to Divide the last R.} \\
 \text{Add } \left\{ \begin{array}{l} 8 = \text{the Cube of } 2. \\ 240 = \text{the Square of } 2 \text{ in the treble Root.} \\ 2400 = \text{the treble Square of the Root in } 2. \end{array} \right. \\
 \underline{\text{Summ} = 242408} = \text{the Subtrahend, to take from the last Ref.} \\
 1 \text{ Remainder.}
 \end{array}$$

7. Here according to the 6th Rule the 2 first Figures of the Resolvend 242, viz. 24 cannot be Divided by the Divisor 126, I therefore put a Cypher in the Root, bringing down the next branch (409) to the last Resolvend 242, and so proceed to form a New Divisor from the treble Root (20), and the treble Square thereof.

V. For Extracting the Root more exactly of any imperfect Cube-Number; you must place 3, 6, or 9, &c. Cypher to the Right hand the Number given, to have the Root Extracted, and so proceed as with the Numbers, and for every 3 Cyphers, one Decimal place must be in the Root as in the Operation following.

Example

The Extraction of the Cube-Root.

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Example 3.

8252.409000000 (10.108

First Divisor 126 }
the 2d = 12060 } 0252409 = the 1st and 2d Resolv. 252 and 252409

6 the treble Root 2.
12 the treble Square of the Root.

Summ = 126 Divisor the first.

Add { 60 the treble Root 20
1200 the treble Square of that Root.

Summ = 12060 Divisor the second.

8 the Cube of 2 last put in the Root.
240 = the Sq. of 2 mult by the last treble Root.
2400 = the treble Square of the Root mult. by 2.

The Summ = 242408 the 1st subtr. to take from the last Resolv.

3d Divisor 1224726 }
4th = 122418060 } 10001000000 { the third and fourth Resolv.
10001 and 10001000000.

606 the treble Root 202.
122412 the treble Square of that Root.

Summ = 1224726 Divisor the third.

Add { 6060 treble Root.
12241200 treble Square of the Root.

Summ = 122418060 Divisor the fourth.

Add { 512 = the Cube of 8 last put in the R.
387840 = the Sq of 8 in the tr. R. (6060)
97929600 = the tr. Sq. of the R. mult. by 8

Summ = 9796838912 the 2d sub. to take from the last R.

204161088 Remainder.

L 2

Note,

*An Ex-
ample of a
Mixed Num-
ber.*

*Notes on
the Work
above.*

Note, From the foregoing Work, That because the first Divisor (126) cannot be had in the 2 first places of the first Resolvend or Dividend, (252) I therefore put (2) in the Root, and bring down the next branch 409; so is the second Resolvend 252409, and the second Divisor whereby to Divide that second Resolvend is the treble of the Root 20, and treble Square thereof 1200, viz. 12060, and the like I have done with the third and fourth Resolvends and Divisors, Cyphers falling in the Quotient or Root.

Note, Also that you might yet Extract the Cube-Root of the last Number nearer the truth, by placing 3 Cyphers to the Right hand the last Remainder 204161088, and so proceeding as in the first Example is directed.

*Cube-Root
of Decimals*

VI. To Extract the Cube-Root of a Decimal, work as you did for a whole Number; but observe,

1. That your Decimals must either consist of 3, 6, 9, or 12, &c. places to be made so, by adding Cyphers to the Right hand, if there be occasion; so that in a Mixt Number, as that in the last Example, a point will always fall over Units place of the whole Number.

*Two Rules
for the
Root of De-
cimals.*

2. For every 3 Cyphers toward the Left hand, next the point in the Decimal given, you must put one Cypher in the Root, and Extract the Root of the Remaining Decimal, as if there were no Cyphers, thus the Cube-Root of .0001728 is .055, and something more; the Root of .00001728 is .025, and a little more, the Root of .000001728 is .0055, and a small matter more, see the Examples.

Example 4.

Cube-Numb.

.000172800 (.055 Cube-Root.

$$\begin{array}{r}
 125 \\
 765 \overline{) 47800} \\
 \underline{15} \\
 75 \\
 \underline{765} \\
 125 \\
 375 \\
 \underline{375} \\
 41375
 \end{array}$$

Remains — 6425

Example 5.

*An Ex-
ample of
the 2 last
Rules.*

The Extraction of the Cube-Root.

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Example 5.

Cube-Numb.

.000017280 (.025 Cube-Root.

$$\begin{array}{r}
 126 \overline{) 9280} \\
 \underline{6} \\
 12 \\
 \underline{126} \\
 125 \\
 150 \\
 60 \\
 \underline{7625}
 \end{array}$$

Remains = 1655

Example 6.

Cube Number.

.000000172800 (.0055 Cube-Root.

$$\begin{array}{r}
 125 \\
 765 \overline{) 47800} \\
 \underline{15} \\
 75 \\
 \underline{765} \\
 125 \\
 375 \\
 375 \\
 \underline{41375}
 \end{array}$$

Remains = 6425

Note,

The Extraction of the Cube-Root.

Note, That I could have made each of these Roots nearer the truth by putting 3 Cyphers to the Right hand of each Remainer, and so proceeding to Extract as before.

C H A P. XII.

The use of the Extraction of the Square and Cube-Roots.

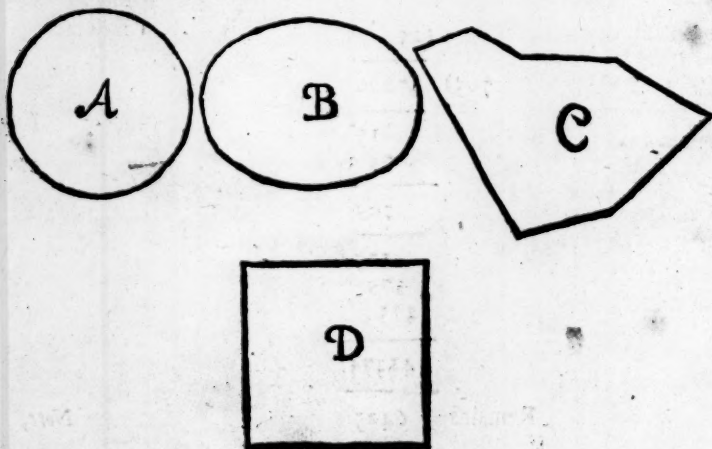
S E C T. I.

The use of the Extraction of the Square-Root.

I. *To reduce any Superficies to a Geometrical Square.*

First use.

IF there were no other use of the Extraction of the Square-Root than this, it were to be much esteemed by all the Students in the Mathematicks: For suppose a Circle as *A*, an Elipsis, or Oval as *B*, or an irregular Multangle as *C*, the Square-Root of the Content of any of them multiplied in it self gives the Content of the Geometrical-Square *D*, which Square is (near enough) equal to any of the other Figures.



II. Another use of the Extraction of the Square-Root is the making *Second use.* that most Excellent Table of Logarithms, useful in all parts of the Mathematicks, which Table is made by Extracting the Square-Root of the Square-Root continually of the Number whose Logarithm is sought, until such time as the Root is so little, that it has as many Cyphers in the Decimal part next the point, as are equal to the Number of places you intend the Logarithms to consist of, &c.

Which Table of Logarithms when with much trouble made (there requiring above 20 several Extractions, and as many Multiplications to make the Logarithm of one Number) is not only useful in Surveying, Dyalling, Navigation, Astronomy, &c. but in the Extraction of the Square and Cube-Roots of Numbers, thereby much facilitating the Work; for to find the Square-Root of any Number by the Logarithms, is nothing but to take $\frac{1}{2}$ the Logarithm of the Square-Number given, and you have the Logarithm of the Root, and the Cube-Root is found by taking $\frac{1}{3}$ of the Logarithm of the given Number, &c. the excellent use of which Table of Logarithms I have only just hinted at, to shew the great use of the Extraction of the Square-Root whereby (among other admirable uses) that Table is made.

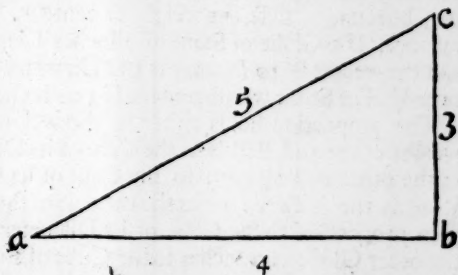
III. A third and material use of the Extraction of the Square-Root *Third use.* is by having any two sides of a Right Angled Triangle as (abc) given, the third may be found, for instance.

1. The Square-Root of the Summ of the Squares of the Lines ab , and bc is equal to the Line ac .

2. If from the Square of the Line ac you take the Square of the Line ab , the Square-Root of the Remainder is the Line bc .

3. If from the Square of the Line ac you take the Square of the Line bc , the Square-Root of the Remainder is the line ab .

These three Cases do answer several propositions in the Mathematicks; and the truth thereof is Demonstrated by *Euclid*, in the 47, 1 of his Elements, for if the base (ab) be 4, and the perpendicular (bc) be 3, the Hypothensé (ac) will be 5, &c.



IV.

Fourth use. IV. By the Content of a Circle, its Diameter may be found by the Extraction of the Square Root, for

As 7854 is in proportion to 1, so is the Area or Content of any Circle to the Square of the Diameter of that Circle, which Square having the Root extracted gives the Diameter; this is demonstrated by *Euclid* in the 2, 12 of his Elements.

There are many more uses; but none so material as those I have mentioned, and that may be gathered therefrom.

S E C T. 2.

The use of the Extraction of the Cube-Root.

I. To reduce any solid body to a Cube; this is performed by extracting the Cube Root of the Content of any solid body; for that Root is the side of a Cube equal to the solid, thus if a long or round piece of Timber, or Stone were 1929781125 Inches solid; the side of Cube equal thereto, is 1245, as appears in the first Example of the Extraction of the Cube-Root.

Second use. II. The content or weight, and side of the Cube or Diameter of any solid being given the side of the Cube or Diameter of any other may be found of different weight or content, in Feet, Inches, &c. for instance, If a Globe of Stone of 9 Inches Diameter, is in content 381, and the weight is 30 l. what is the Diameter of another Globe of the same kind of Stone, whose content is 500 Inches, and the weight 39 ³⁶⁹/₁₀₀.

The proportion holds either by the weight or content, for as the weight of the 30 l. Ball is to the Cube of its Diameter; so is the weight of the other Ball 39.369, to the Cube of its Diameter, whose Cube-Root is the Answer; or as the content of the Globe 381 Inches solid, is in proportion to the Cube of its Diameter 9; so is the content of the other Globe 500 Inches to the Cube of its Diameter, whose Cube-Root is the Diameter sought.

The truth of this is demonstrated by *Euclid* 18, 12 Elements, and by the same Reason the measure of long, round or square solids, or the Concavity of Guns, &c. being compared are easily discovered.

F I N I S.

